IMPARTING CREATIVE KNOWLEDGE-GENERATION PROCESSES ACROSS CURIOSITY-DRIVEN DISCIPLINES: ACCOUNTS OF PURE MATHEMATICIANS, FINE ARTISTS, AND PHYSICAL SCIENTISTS

by

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Abstract

This research queries how the creative knowledge-generation processes in the curiosity-driven disciplines of pure mathematics, the physical sciences, and the fine arts are imparted and developed, with attention to the behaviors and dispositions of the process and to disciplinary aesthetics and affect. An introduction frames this research in a conversation about how curiosity-driven pursuits are central to liberal education and offers a definition of the creative knowledge-generation process that is curiosity-driven, exploratory, transformative, iterative, and ongoing. This conversation connects the aims of liberal education, creative processes, and curiosity-driven pursuits. Then, a theoretical paper investigates the accounts of mathematicians and artists to compare the processes in these creative disciplines, noting that they share similar behaviors, dispositions, and aesthetic considerations, and that they are socially constructed, asking why such similar processes are often viewed differently. Next, two studies drawn from semi-structured interviews with faculty members to ask how they developed their creative knowledge-generation processes and how they impart those processes to students. The first of these studies looks to interviews with nine mathematicians and discusses the ways that question-finding is learned and practiced socially, noting the role of mentorship; issues of social capital; and the development of aesthetic values. The next study looks to 16 interviews with faculty in the fine arts and physical sciences to query how the processes are often imparted and developed informally in the social environments of the studio, lab, or field, through mentorship, middle-mentorship, or peer-learning. Findings also indicate issues of in-groups and out-groups and gender biases. A discussion chapter looks at how social capital, cultural capital (in the form of disciplinary knowledge and aesthetics), curiosity, and wonder interact with the ways the creative knowledge-generation process is imparted and developed in these disciplines.

Keywords: creative knowledge-generation process, liberal education, curiosity, inquiry, higher education, mathematical process, artistic process, aesthetics

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Introduction: Curiosity, the Creative Process, and Liberal Education

As the aims of higher education are debated, new initiatives and educational formats emerge that often minimize curiosity-driven pursuits in the liberal arts. While watching programs, schools, and larger discourses on higher education shift, I have questioned how the value of curiosity-driven pursuits can be articulated and potentially preserved, viewing these pursuits as intrinsically important parts of the human experience and of a liberal education, and recognizing the creative behaviors they engage and the intellectual and affective experiences they afford to students. As part of this broader consideration, this project asks how the creative knowledge-generation processes of these curiosity-driven pursuits are developed and imparted in higher education.

The original purpose of a liberal education, to engage the intellectual behaviors of liberated thinking and attuned perception (i.e., Budwig, 2018), was intended to be achieved via study in the “arts,” comprising the trivium of humanistic disciplines and the (nowadays often omitted) quadrivium of mathematical arts: geometry, arithmetic, astronomy, and music. The study of these curiosity-driven liberal arts disciplines serves as a platform for the development of liberated thought by facilitating the ability to create new knowledge and to flexibly consider complex ideas from multiple perspectives. This includes the ability to perceive, formalize, and investigate new questions. Truly liberated thinking chooses what to ask.

Looking to Dewey (1934/1980), this research regards liberated thought arising from an interaction of the individual with the external environment as a creative act, resulting in the creation of a new idea, new knowledge, or new awareness. The new idea arises from “a ferment,” “a doing and an undergoing” in an encounter with the phenomenon of interest–the emerging artwork, the natural world–which is a transformative interaction of perceiving and formalizing, of becoming aware. The transformation that happens through this exploratory encounter, which is motivated by curiosity, affects both the explorer and the explored, changing them both through the creation of new meaning. Dewey says of this experience, “Attitudes and interests are built up which embody in themselves some deposit of the meanings of things done and undergone. These funded and retained meanings become part of the self” (275). And this connects the purpose of a liberal education, the creative process, and the engagement in curiosity.

In the transformative, exploratory encounter, the individual pursues perception via an intrinsically motivated “plunge” that requires directed energy to experience and engage with that which is perceived. Dewey discusses an inner impulse to interact in this way, an intrinsic motivation to attend, engage, and to be open–a curiosity. This intrinsically motivated plunge results in the creation of new meaning, a creative knowledge-generation process. In this way, Dewey’s model of doing and undergoing is a curiosity-driven process of creative knowledge-generation, and perception is part of Dewey’s description of the transformative experience in the same way as attuned perception can be seen as a goal of liberal education; in both cases this perception drives inquiry.

This research regards curiosity-driven disciplines as those that have largely evolved in response to such an intrinsic motivation to engage for the purpose of the experience with the ideas or phenomena of the subject. As Lockhart (2002) explains about mathematics, it is a human endeavor valued for its own sake. Similarly, art is often practiced “for art’s sake,” and the physical sciences evolved to entertain a quest to understand the physical world. These investigations for the sake of curiosity engage this transformation towards new awareness, the generation of new knowledge.

The working definition of the creative knowledge-generation process used in this research derives from studies with, and accounts of, fine artists, pure mathematicians, and curiosity-driven scientists and is inclusive of Dewey's description of doing and undergoing, recognizing a similar texture of exploration and discovery in these accounts. It is worth noting, though, that researchers interested in the creative knowledge-generation behaviors of practitioners across disciplines often look to artists to study how this process generally unfolds. It is also important to note that this present inquiry regards the investigative and inquiry behavior of practitioners rather than qualities of the persons themselves, drawing a distinction in the locus of creativity used in this discussion: the behaviors rather than the individual. The behaviors of the creative knowledge-generation process can be seen as those by which practitioners endeavor towards a novel ideas within their discipline, though another way of looking at the behaviors of this process would question the role of the endeavor *towards an object* and would introduce instead a way of engaging openly as to invite iteratively new ideas: a texture of exploration and discovery that creates new knowledge in a durational, ongoing way.

A definition of this texture would be explained by Dewey's (1934/1980) model: the act of creating new knowledge within the artistic process is an encounter with world, an interaction between a person and some aspect of their environment, tangible or intangible, including the social environment, its traditions, and its institutions. Dewey's description of an encounter with the world includes encounters with its influence on the individual. As such, "the real work of art is the building up of an integral experience out of the interaction of organic [of the subject] and environmental conditions and energies," which, according to Dewey, should conclude in a feeling of harmony arising from the restoration of a lost integration with the environment (p. 67). This description of the artistic process, which Dewey relates to the sciences in later works (see Hallman, 1964, for a concise elucidation), also addresses the perception and integration of multiple perspectives pursued via a liberal education. The integration of the individual with the environment or phenomenon of interest, via the active plunge into perception and the iterative creation of meaning, requires openness to the experience that phenomenon presents– an openness that allows for an experience beyond previous categorization. This integrative experience, seen as the *real* work of art, which is then related to experiences with natural phenomena in the sciences, is what unfolds in the creation on new knowledge via inquiry and exploration.

Whether the creative process be seen as a way of engagement, exploration, and discovery towards iteratively novel ideas, or the means by which a defined object or product is created, the practitioner first takes a stance of perceiving, questioning, and interpreting, a stance of curiosity. The practitioner then attends to the idea and engages in iterative inquiry, investigation, reflection, interpretation, and revision, within a disciplinary epistemology, arriving at iterations of new knowledge. This process does not happen in a set sequence, nor does it necessarily aim for a particular goal. The choices in these behaviors are guided by affect, aesthetic sensibilities, and wonder. Whether in math (i.e., Halmos, 1993; Ervynck, 1991), the arts (i.e., Mace & Ward, 2002; Chemi, Jensen, & Hersted, 2015; Fortnum, 2013) or the sciences (i.e., Cuzzolino, 2019; Dayton & Sala, 2001), the ongoing investigation, experimentation, and reflection of this iterative, creative process require an intentional openness to create new meaning and knowledge and motivation to persevere in an open-ended investigation.

The present investigation then asks how creative knowledge-generation processes, including inquiry and exploration, are imparted and developed in the curiosity-driven disciplines of pure mathematics, the physical sciences, and the fine arts, with attention to the dispositions of the process, affect, and disciplinary aesthetics and epistemologies. This research also extends the conversation about the values of liberal education, a conversation that often addresses only the humanistic disciplines, to all those disciplines that primarily engage a curiosity-driven process of inquiry motivated intrinsically by a desire to explore and experience.

Budwig (2018) says, “Typically, the study of liberal education and the research university are distinct,” discussing historically different views of knowledge and practice in each, but the formulation of a question and the investigation of it, often in a community of university research scholars with shared curiosity, can perhaps also be seen as one of the academic behaviors most clearly connected to the liberated thought of a liberal education; hence, this research queries creative knowledge-generation processes, or research processes, in liberal arts disciplines as practiced at colleges and universities.

Summary of Included Papers

This research is presented as three papers. The first paper compares accounts of artistic and mathematical processes, often written by educators and professionals in the disciplines to explain what they feel is a process unrecognized by others. The resulting theoretical paper discusses the similar arc of investigative behaviors these two disciplines share, including idea generation and iterations of investigation, reflection, and revision. Because these are the curiosity-driven processes of fine art and pure mathematics, they each include the artist’s or mathematician’s choice of a question to pursue. These processes similarly emphasize dispositions of openness and perseverance, rest similarly on disciplinary aesthetics and epistemologies, and respond and contribute to the social construction of the disciplines. This comparison provides a platform for discussing the similarities and differences in the ways these disciplines are taught, raising questions about how similar practices are taught differently and how these educational differences affect the invitation to participate in the disciplines.

Building on this theoretical research, I conducted semi-structured interviews with faculty members in pure mathematics, curiosity-driven physical sciences, and the fine arts to better understand how they developed their processes and how they impart those process to their students, inclusive of their dispositions and aesthetic values, noting cross-disciplinary similarities and differences. Using a subset of nine of these interviews, the second paper discusses elements of the processes of mathematicians, primarily the ways question-finding is learned and practiced socially, noting issues of social capital and the development of aesthetic values as they play into this learning and practice. These experiences occurred almost exclusively in graduate school. The third paper draws on interviews with 16 artists and scientists and discusses the informal learning experiences that occur in the social spaces of the lab, studio, and field, reflecting on how the social environments created in those spaces, including the mentorship and peer interactions that occur, shape the way the processes are imparted. Here, the artists' accounts describe arriving at creative inquiry and authorship in their undergraduate studies, while the scientists mostly discuss graduate school.

Starting from the idea that engagement in curiosity-driven pursuits is integral to the aims of liberal education, this research potentially contributes to a conversation about how students are taught or mentored towards curiosity-driven creative knowledge-generation processes, the iterative and ongoing inquiry, exploration, and discovery of them, and the authorship of questions and new knowledge. It may begin to distill particular ways pedagogical and environmental factors invite and foster participation in such learning. It also may contribute to a conversation about the way the invitation to trust one's curiosity, and to ask questions based on it, can be equitably extended such that all students (and people) can have the opportunity to participate in the creation and co-authorship of these disciplinary conversations and the culture they influence.

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Paper 1: A Comparison of the Creative Processes of Pure Mathematics and Fine Art

Abstract

This theoretical paper discusses the similar arc of investigative behaviors in fine art and pure mathematics, including idea generation and iterations of investigation, reflection, and revision. It compares accounts of artistic and mathematical processes, often written by educators and professionals in the disciplines to explain what they feel is a process unrecognized by others. These processes similarly emphasize dispositions of openness and perseverance, rest similarly on disciplinary aesthetics and epistemologies, and respond and contribute to the social construction of the disciplines. Because these are the curiosity-driven processes of fine art and pure mathematics, they each include the choice of a question to pursue, which affects the role of dispositions and aesthetics. The similarities of these processes then raises a question about the ways the disciplines are differently regarded and taught, and how this teaching affects the way students are invited to participate in inquiry and investigation in fine art and pure math.

Keywords: creative process, mathematical process, artistic process, curiosity, disciplinary aesthetics

Paper 1: A Comparison of the Creative Processes of Pure Mathematics and Fine Art

As higher education rapidly changes and the aims of education are debated, the disciplines once included in the liberal arts diminish while other courses of study, based on a different view of the purpose of higher education, expand. Meanwhile, there continues to be a good deal of discussion about educating students towards creative behavior, and the programs and models brought forth to address this goal present many different interpretations or definitions of such behavior. Programs across the fine arts, certain sciences, and applied fields suggest they address creative processes, yet the evaluation (and valuation) of such programs differ as they may encourage idea generation, self-expression, or problem solving. Furthermore, some curiosity-driven creative disciplines, like pure mathematics, are less often included in these discussions. But mathematics has long been understood to be a creative discipline, particularly by those who practice it, and it is a cornerstone of the liberal arts, comprising two of the original seven "arts," arithmetic and geometry.

This research draws a connection between the full creative process, inclusive of idea-generation, and the exercise of curiosity-based inquiry. As curiosity-driven, creative disciplines, pure math and fine art exercise the liberating intellectual behaviors that are hallmarks of a liberal education: to flexibly consider complex ideas from multiple perspectives and to perceive, formalize, and investigate new questions (i.e., Budwig, 2018). How is the creative process of making new mathematical knowledge imparted, and how is the education towards creative mathematical behavior similar to that of art, a field more readily recognized as creative? How do educational practices in these disciplines invite participation in the skills of creative, curiosity-driven authorship, and what are the implications for the aims of inclusive liberal education and the authorship of new ideas?

In order to investigate how creative knowledge-generation processes are imparted, it is important to understand what these processes look like in practice, how they are similar and how they differ, and to question whether apparent differences are due to differences in the processes themselves, to disciplinary foundations, or to the way the work is presented in society. This paper compares creative knowledge-generation processes in fine art and pure mathematics, anchoring this comparison in descriptions of the artistic and mathematical processes from their own disciplinary literatures. By elucidating the connections between these processes, it is clear that they follow similar arcs of behaviors, that they engage similar dispositions, and that they rest similarly on disciplinary aesthetics and epistemologies. This comparison then provides a platform for discussing the similarities and differences in the ways these disciplines are taught, raising questions about the ways these similar practices are taught and perceived differently, and how educational differences affect the invitation to participate.

Pure mathematics and fine art are chosen from among the curiosity-driven liberal arts disciplines due to the detailed descriptions of the processes written from within the disciplines themselves, emphasizing their creative nature. Interestingly, these descriptions are often written to explain what the mathematicians or artists feel is a process unrecognized by others, who might make inaccurate assumption about their work. As artist and art theorist Graeme Sullivan (2010) explains, "…what artists do is mostly misunderstood," which is clearly echoed in Paul Lockhart's (2002) lament, "… nobody has the faintest idea what it is that mathematicians do."

When describing the sources of ideas, the act of investigation, or the attitudes required and developed in the process, mathematicians and artists may describe similar phenomena somewhat differently, though distilling their descriptions reveals closer connections in the experiences of creating art and mathematics. And in both cases, the processes are more than the creation of the product; they are ways of knowing and ways of pursuing and experiencing that knowing. A comparison and deconstruction of the behaviors of the process also avails of asking how disciplinary aesthetic values and epistemologies play into these behaviors, clarifying where the processes are similar *with respect* to their values, and where the values and disciplinary stances may change the process itself.

Behaviors of the Process

Descriptions from within each discipline show that the processes of artists and mathematicians include the behaviors of idea generation, investigation, reflection, and revision. Because these are the curiosity-driven processes of fine art and pure math, as opposed to design or applied math, they each include the artist's or mathematician's choice of a question to pursue, rather than an address of a question determined a priori. This question-finding behavior includes the choice of content and the initial method of approach. Both the method and content of the primary question may be revisited and revised to differing extents within the process, sometimes entirely changing the question.

This idea or question is then investigated through actions to transform, describe, articulate, model, or illustrate it. These efforts often include attempts to relate or connect ideas or to try out certain examples or materials. Burton (1984) describes mathematical thinking as a *style* of thinking, as contrasted to thought about the discipline of mathematics, arguing that mathematical thinking can be applied to any content, though noting that "questions of a mathematical nature might more readily expose such thinking" (p. 36). She outlines this type of thinking as the study of relationships via such tasks as developing correspondences or equivalences between ideas, and ordering, combining, or making substitutions to transform them.

Much of this is done as an experiment, an effort to try some approach to the idea or question to see what happens and what that activity will reveal or teach the artist or mathematician (i.e., Chemi, Borup Jensen, & Hersted, 2015; Ecker, 1963; Mace & Ward, 2002; Sjöholm, 2013; Schwab, 2015; Sriraman, 2004; Wenzel, 2018). In response to interviews with visual artists, Sjöholm (2013) notes three elements of experimentation within the artistic process: self-direction, reflection, and elaboration. Two of the four parts of Mace and Ward's (2002) model of the artistic process, idea development and making, emphasize the experimental nature of the process. Fortnum's (2013) perspective relates these experimental behaviors to artistic ways of knowing: thinking and knowing in a medium *requires* an experimental and responsive approach.

Experimentation may involve the creation of abstract or tangible objects or models to explore ideas, such as invented mathematical constructions or conceptual objects that can be manipulated with mathematical tools to test their properties, or the creation of an artistic object, a "process piece" that can serve as a site for exploring relationships or working out ideas. Drawing on interviews with five tenured mathematicians, Sriraman (2004), also a mathematician, describes several heuristics mathematicians use to test ideas, solve problems, seek patterns, verify consequences, and infer from analogy using examples and counter examples. Looking to the accounts of Polya, who reported in 1954 that mathematicians use a variety of heuristics to look at and manipulate situations in different ways, Sriraman summarizes that these heuristics are decision-making devices meant to help clarify the mathematician's options as they engage in transformations to explore relationships between ideas.

Wenzel (2018), a philosopher and mathematician, describes how he constructs models allowing him to work with their specific properties, make changes to the situation, and immediately view the results of the changes, potentially illustrating what happens in general or in particular cases. He describes that the models that are manipulated do not need to be fully developed ideas, adding that "underdetermination can be a good thing" when playing with models, so new ideas can arise and indicate potentially surprising qualities (p. 324).

Looking at the development and employment of heuristics in this way, as tools to investigate ideas, points to certain similarities and differences between mathematical and artistic processes. On the one hand, mathematics is the creation of ideas, whereas studio art does usually generate an object-based product alongside, or as the articulation of, the idea. Therefore, the creation of in-process heuristic objects in the mathematical process seems notable as these objects are even less often seen than the final products themselves (which are viewed primarily within the discipline). But when defining a heuristic in this way– as a more tangible, transformable object in the service of exploring an abstract idea, even if the object is a mental diagram or construction– then the creation of any preliminary sketches, models, photograph collections, or written thoughts in the artistic process would fit this definition, and engaging with them, as artists do, would befit Wenzel's description of modifying heuristic objects to learn from the results even when, or particularly when, the models themselves are not quite complete. Meanwhile, drawing on Borgdorff's (2011) idea of the art object as an epistemic thing– an object that both presents new knowledge and *is* the knowledge itself– the entirety of creating an art object can be seen as the development of a heuristic, a site for exploring the developing idea.

The artist or mathematician makes many responsive choices within this process of investigation, which result from reflection and often lead to revision of the method or question. Reflection can be seen as a both a deliberate moment aside from action and as part of the ongoing behavior of investigation, an approach of reflecting within investigating, such that the practitioner notices and makes momentary decisions, adjusting the methods of investigation in response to the emergent relationships and outcomes along the way.

Descriptions of artists' practices include either explicit or implicit activities of careful evaluation or contemplation (i.e., Walker, 2004; Sjöholm, 2013; Detlefsen, 2012). Walker (2004) primarily describes art making as a *reflective practice*, and Sjöholm (2013, 2014) describes how reflection in the artistic process relates to experimentation and to the transformation of ideas into artwork, illustrating the role reflection continually plays in the experimental actions of investigation. Wenzel (2018) describes how, for the mathematician, "contemplation and creation constantly interlace and reflect each other." These accounts point to practices of both artists and mathematicians in which a reflective approach is necessary in order to thoughtfully respond to the outcome of momentary experiments.

This process of reflection, revision, and investigation often happens nonverbally; instead, the practitioners think in images or in their medium or symbol system. Ecker (1963) describes reflection and revision within the actions of art making as "qualitative thinking," which he describes as momentary thinking in the qualities of the evolving artwork. The artwork presents a quality that the artist fluidly addresses in the moment with a certain method or material, without articulating this response in language. Similarly, mathematicians responsively manipulate a mathematical object using a particular approach or technique, thinking in the mathematical symbol system or images, and responding to the discovered properties or qualities of the mathematical object or idea nonverbally.

Inquiry in these processes is an iterative and durational behavior, as idea generation and investigation lead, through reflection, to revision. As a result of a deliberate reflective pause or reflection-in-action, the revision that follows may change the tools or methods of investigation, may potentially reframe the investigation (i.e., from examining special cases or particular materials to creating a diagram or relating images), or, in some cases, may revise the initial question itself. Silver (1997) describes mathematical creativity as the interplay of flexible problem-posing and problem-solving. This resonates with the account of Halmos (1968, p. 178), who describes the mathematical process as one of iteration, illustrating that the creative knowledge generation is not the solution but the reframing along the way:

> Mathematics is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas and he becomes convinced of

> their truth long before he can write down a logical proof. The conviction is not likely to come early– it usually comes after many attempts. It often happens that months of work result in the proof that the method of attack … cannot possibly work, and the process of guessing, visualizing, and conclusion-jumping begins again.

Jones (2010) argues that iteration is necessary for artistic creativity, which he defines as a transformation that amounts more to a reframing of a problem than solving one. The transformative reframing arises due to iteration and can be described as a change in paradigm, state of mind, or way of being. Applying for a moment Jones' definition of the creative process to mathematics, then the iterative reframings of both mathematical and artistic research seem more closely connected, as each iteration of investigation, reflection, and revision reframes the project, creates new questions, and creates a new platform upon which new understanding is sought.

Dispositions of the Process

Another component of these behaviors is dispositional. Whether in art or math, the unpredictable process of engaging in ongoing negotiation or dialogue with the developing work requires and further develops certain attitudes or states of mind: curiosity, an openness to new ideas and approaches suggested by the evolving work, comfort with non-closure and open-ended problems, motivation, and perseverance in the task (i.e., El-Sahili, Al-Sharif, & Khanafer, 2015; Mann, 2006; Karakok et al., 2015; Chemi et al., 2015).

Openness allows for idea generation and reflective investigation. In his discussion of mathematical creative processes, Ervynck (1991) depicts this openness, describing an essential "state of mind prepared for mental activity that relates previously unrelated concepts" (p. 44). Chemi et al. (2015) summarize the approaches of the artists they interviewed, saying that they practice an *intentional* openness, which also relates to gut feeling, intuition, reflection, and to the self-awareness and agency to choose such openness. Pursuing new connections and relationships through experimentation also involves creative risk, and a particular kind of open-mindedness is required to engage in these explorations knowing that they might lead to an unpredictable realization or require greater revision or a new approach (i.e., Mann, 2006; Wenzel, 2018; Gude, 2004, 2007; Dewey, 1934/1980).

The openness required can also be durational. Within Mace and Ward's (2002) model, artists often delay closure in the process, staying open and avoiding a conclusion while revisiting ideas and letting the evolving artwork unfold. This speaks to a nonverbal or preverbal acknowledgement of the ideas being explored, a knowing in the materials but not yet translated to concrete explication or language. Walker (2004), observing the processes of her university students, says that those who delay closure in the process, allowing the artwork to evolve without determining what it should be, create more successful artworks. This resonates with Wenzel's (2018) appreciation of the pursuit of vague ideas in mathematical experimentation; those vague ideas allow for new perspectives and possibilities as the mathematician transforms and manipulates them, in contrast to ideas already made concrete that would behave more predictably and yield fewer surprising new directions.

Whereas the openness discussed above can be seen as a disposition towards action within the process, Sjöholm (2013) says intuition and improvisation are *skills* of the artistic process, though she says these skills are based on practical experience, which raises the question of how to regard this openness, as epistemology or as strategy, as disposition or skill, and how it can be observed or taught. Relating these ideas, in particular Sjöholm's assertion and Mann's (2006) description of mathematical creativity arising from a frame of mind that can be fostered, the literature across both disciplines indicates that the dispositional qualities of the process are learnable behaviors that increase with experience.

This durational and open-ended process, with new ideas emerging as the artist or mathematician responds and attends to the evolving work, requires a commitment to staying open as plans change. Essentially, inquiry into an unknown content via an iterative process of investigation, reflection, and revision implies perseverance in a process that may take any amount of time and that does not necessarily follow pre-set rules. Both mathematicians and artists respond to experiments that offer new insights, and to those that do not work. They then keep going, regardless of frustration, trying more ideas and adding to their knowledge through multiple approaches. Chemi et al. (2015) describe the "motivation, resilience, and persistence" involved in artists' accounts of artistic discovery and research (p. 136). This perseverance is also made palpable in Halmos's (1968) recounting of the process, offered above, in which months of work may result in starting over.

Artists may develop strategies to maintain the perseverance required to engage with the openness of the artistic process by setting parameters regarding breaks, materials, or evaluating artwork. By determining guidelines that their investigations may take a certain amount of time, transpire in a given set of materials, or be evaluated for certain criteria or at particular times, the artists provide a structure so they may focus. The parameters allow them to persevere in attending and responding to the openness of the process within a structure that gives them some way of managing the encounter with evolving artwork (O'Grady, 2017). Mace and Ward (2002) allude to something similar, recognizing that artists may discard a work of art within the process, or may revisit one, taking breaks from certain pieces while leaving their options open as they manage their attention to evolving artworks. Mathematicians also describe the need to attend the potentially frustrating encounter with yet-unresolved mathematical ideas via self-sustaining habits; many mention the sorts of breaks they take, either from a given project by moving onto something else, or from work more generally. For example, Burton (1999), reporting on interviews with mathematicians, notes that they often structure breaks to manage the encounter with their durational, open process. These accounts indicate a degree of self-knowing in order to assess and respond to one's own socio-emotional state.

The behaviors of the process involve a great deal of choice, which requires a sense of personal agency to do the choosing, whether the choice be about the initial question, a starting point to investigate it, revisions in method or content, or the choice to engage at all. Artist and mathematicians must entitle themselves to think independently, to entertain their curiosity, to value their questions, and to commit to an exploration of those questions. This sort of self-regard is necessary to engage in the pursuit of new ideas and to stay open to the evolving process throughout iterations of inquiry. These behaviors then relate to goals of liberal education, which require sensitively attending to one's curiosities to inquire about oneself, the world, and the relationship between them.

Epistemology and Aesthetics

Each discipline has an epistemology, a filter through which perception occurs and new knowledge is formalized. This epistemology is developed and evolves along with the discipline's canon and methodologies, and it underscores and is embedded in its symbol system, tools (materials and techniques), and formats. Therefore, the tools and languages of mathematics and art also bring into contemporary practice their historical values and traditions of the discipline. The mathematical symbol system and grammatical organization of information imparts a prioritization of certain types of knowing and meaning-making. Artistic materials and formats, through which information is generated and expressed, similarly impart a prioritization of certain ways of knowing unique to art. Engaging in the processes of these disciplines, and in their nonverbal methods and presentations, involves perceiving and formalizing new insights through the disciplinary epistemological filters that determine what can be "known" or viewed as knowledge.

The artistic process is often described as a way of knowing or as an artistic epistemology, as artists formalize knowledge via artistic inquiry, often in in nonverbal means (i.e., Holert, 2009; Fortnum, 2013; Schwab, 2015; Sullivan, 2010; Richardson & Walker, 2011; Borgdorff, 2011). Borgdorff (2011) discusses how artworks serve both as epistemic things, the things through which knowledge is produced, and the knowledge itself. As they evolve throughout the process of their making, these epistemic things present questions, serving simultaneously as the site of the evolving idea and as the expression of the idea. In other words, the interaction with the artistic medium is a dialogue that gives rise to the new knowledge, allowing for certain ideas to emerge in the negotiation with the materials. This new knowledge is therefore present, and presented, in the art object. This way of knowing through the creation and interaction with the art object as an epistemic thing resonates with Ecker's (1963) description of an artistic way of thinking in a medium– in short, the material mediates the knowing.

The choices and iterations of the mathematical creative process are based on a mathematical symbol system, with its grammar and way of formalizing knowledge. This is language, according to Halmos, "is a precise and subtle language designed to express certain kinds of ideas more briefly, more accurately, and more usefully than ordinary language" (p. 178). The symbols index ideas of a certain complexity, and as those ideas are referenced, connected, and arranged, the mathematician can investigate the relationships among these complexities. Ervynck (1991) says that the symbolic representation of ideas in the mathematical language allows for a "condensing" of ideas that makes the relational and selective work of the process possible. Thinking then in the symbol system of mathematics, with the ideas condensed or indexed in its symbol system and arranged in its grammar, allows of certain kinds of relationships to emerge and become known much more efficiently, and more completely, than relationships of another kind.

Aesthetic values are inherently part of disciplinary epistemologies and guide the processes in both disciplines, affecting the selection of questions, the regard for and response to emerging information, and the evaluative choices in revision. That artistic practice involves aesthetic consideration needs no explanation. The arts are understood to model aesthetic values, and therefore aesthetic values are not explicitly mentioned in many discussions of the artistic process, though they are implied as guiding choices in art making. When aesthetics are discussed with respect to the arts, many different sets of aesthetic values come to the fore; the aesthetic values of different cultural contexts have different emphases. Contemporary art rests in a multitude of differing aesthetic values, including aesthetics of beauty and aesthetics of relational interactions. Still, the role of these aesthetic values is the same: they underscore choices of content, concept, and form at the outset and during the iterative process of reflection and revision.

Throughout the literature on mathematical processes, frequent mention is given to the beauty and elegance of mathematics. As Sinclair (2004) summarizes, "[r]ecognition of the beauty of mathematics … is almost as old as the discipline itself" (p. 263). Like with the arts, the aesthetic values of mathematics as a discipline, and the aesthetic preferences of the individual mathematician, guide the choice of content and the ongoing choices of the mathematical process. Typically, mathematical aesthetics are discussed as aesthetics of beauty, with notably less variation than the aesthetic values underscoring contemporary art. This aesthetic of beauty is present in the preference for resolutions in which complex, abstract ideas become refined to elegant and simply-stated expressions. Mathematicians discuss their "beautiful theorems, elegant proofs, cute short cuts, powerful symmetries, not to mention the architecture of the structure they are building" (Dienes, 2004). Illustrating this beauty of ideas, Hardy (1940) insists that, "[t]he mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas … must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics" (p. 14).

Like with the artistic process, where aesthetic sensibilities are understood to inform the choices of the process, aesthetic values influence iterations of mathematical inquiry. Comparing mathematicians to artists, Wenzel (2018) elaborates that both make "constant momentary aesthetic judgments" based on their taste preferences, allowing their aesthetic sensibilities to inform what changes or ideas to select, develop, or discard throughout the process of investigation, reflection, and revision (p. 325). Hadamard (1945/1954) explains that a sense of beauty guides mathematicians towards effective combinations of ideas by indicating the option most demonstrably connected to the aesthetic values of the discipline, a notion that the aesthetics of the solution will relate to the aesthetics of the whole, which is elaborated by Whitcombe (1988), who says that the aesthetic sensibilities of mathematics allow the ideas generated by speculation and guessing to be productively selected.

The aesthetic experience also motivates sustained engagement in the mathematical process. For instance, Karakok et al. (2015). reporting on interviews with six mathematicians, found that almost all mentioned aesthetics as elements of their processes; the quotes offered in the paper suggest that the mathematicians were looking for an aesthetic way of working. El-Sahili et al. (2015) describe the experience of perceiving unexpected links among mathematical ideas during the process as one way to experience mathematical beauty. While the aesthetic sensibilities of art and mathematics may privilege different qualities, the aesthetic experience similarly motivates and guides practitioners in both disciplines in their work.

Recalling that Halmos drew a close comparison between math and painting in 1968, he stated that "modern painting and modern mathematics are far out– too far out in the judgment of some," discussing each as a navigation of ideas in abstraction, and noting that deciding how close to reality either discipline should be is a delicate matter of judgment. At the time, this summary drew a clear comparison between a main aesthetic consideration in art, whether it was getting too far out, as well as in Halmos' experience of mathematics. Furthermore, this comparison anchors an aesthetic comparison in increasing conceptual abstraction in both disciplines. It was made during a historical moment when formalist abstraction in painting was among the most celebrated artistic formats, and it entertains that there is something deeper guiding the formal concerns. Indeed, the concerns of painters and art theorists at the time extended beyond the formal, looking to the ways that these practices related to the definition of art and to philosophical or cultural questions of the time. The aesthetic sensibility of the artists was about a fit to a philosophical or theoretical stance, much as it is in math.

While this comparison was fitting at the moment, some contemporary comparisons of artistic and mathematical aesthetics look to a past moment in art rather than comparing the work of contemporaries in both disciplines. These discussions often describe a painting process in which the artist arranges shapes and colors, referencing Impressionism or Modernist abstraction, even as more diverse formats of contemporary art have become mainstream. And while such accounts do underscore similarities in the ways aesthetics guide the processes painters and the mathematician, these formal comparisons without consideration of the relevant context or disciplinary purpose of the formal choices narrows the perspective on the similarities between art and math and creates a potential confusion. Taking a broader view of contemporary practices in both disciplines invites more salient comparisons of the ways that aesthetics underscore choices of content and methods throughout the process and relate to their disciplinary epistemologies and evolving traditions. Such a comparison also highlights the ways aesthetic values relate to the open questions and current state of each discipline as artists and mathematicians work to evolve the conversation (and canon) forward.

Ultimately, as Lockhart (2002) puts it, the role played by aesthetics illuminates the ways these disciplines are human pursuits, given meaning because they are simply that– human endeavors valued for their own sake. Both art and math have responsively evolved their canons, epistemologies, and methodologies, where the exercise of creating new ideas in either of these disciplines is an aesthetic one in which the epistemological way of knowing in the discipline imports into the process a set of aesthetic considerations that motivates and guides the creation of an aesthetically fitting new moment in the conversation.

Socially Constructed Disciplines

While there have historically been arguments that both of these pursuits are based on universal truths, pure mathematics and fine art are socially-constructed disciplines that follow a trajectory of evolving canons, methodologies, and values. Art is an essential human activity spanning all cultures– a universal with many traditions of personal and cultural expression and exploration. The *institution* or *academic discipline* of art, however, can be seen as a longstanding conversation that is a part of this human activity and reflects upon it. The conversation continually evolves to entertain new perspectives about the definition of the discipline of art and its importance in a given context. The social construction of art as a discipline is fairly transparent when looking at any art history text or art museum: the types of artwork celebrated clearly change over time. In fact, much of contemporary art plays at the edge of, or intentionally queries, the definition of art itself.

Within the discipline of mathematics, some have historically suggested that mathematics exists independently of human creation; taking a Platonic view, they suggest that it is absolute, and mathematicians discover it rather than create it (i.e., Hardy, 1950; Hadamard, 1945/1954). Taking a different view, others suggest that mathematics is a human construction with a social history determined by the questions posed by previous mathematicians (i.e., Burton 1984, 1999, who also discusses Lakatos, 1976). Therefore, the practitioners' interests guide the discipline. Whether the subject of mathematics is seen as a set of absolute natural objects and relationships, or as human-created constructs and ideas, the *human exploration and articulation* of those relationships and of the properties of mathematical concepts and objects creates the discipline of mathematics.

As these are both socially-constructed disciplines, with the questions asked in one moment standing on what has been asked, developed, created, or demonstrated before, the canon has been determined by previous practitioners' questions and curiosities, which were guided by their contextualizing sets of values. The disciplinary conversations then implicitly assert a set of canonical values that influence what would make for relevant and interesting new math or art. And while the project of art seeks more directly and overtly to query the parameters of the discipline itself, the canons and values of both disciplines determine not only the relevant starting points of the process, but also the evaluative choices within reflection and revision.

The canon, methodology, and epistemology of each discipline then evolve together, influencing each other, as practitioners use and develop tools to investigate open questions and produce new knowledge that creates the next moment of the conversation. The artistic and mathematical processes, then, are canon-making processes. The values that underscore the moments in the discipline's histories are present in the ways of knowing, and the ways of learning and teaching them.

Learning the Processes

Behaviors

Given that the artistic process and the mathematical process, as practiced by professional researchers and knowledge-makers in their disciplines, are similar in terms of behaviors and dispositions, and that they are both socially-constructed fields that similarly rely on disciplinary aesthetics, epistemologies, and canons, why are these disciplines generally seen so differently? Art is often viewed as the prime example of a creative and aesthetic field and is studied as an exemplar, while mathematics is frequently viewed as linear and rigid, even while the accounts of mathematicians describe the role of experimentation, guesses, and beauty in a similar creative process.

One clear contributor to this perceived difference lies in the way these disciplines appear in schooling. The traditions of art education emphasize experimentation, play, self-exploration, and expression. Art class engages dispositions of openness and curiosity and directly addresses aesthetic values. Traditions of mathematics education emphasize accuracy, mastery of algorithms, and symbol system fluency. A primary distinction is that math teaching rarely asks students to come up with their own ideas or questions (with the exception of inquiry-based learning), while this is a frequent part of art class from the beginning. In this way, the students are authors of their own art. Katz and Thoren (2017), in discussing inquiry-based approaches to mathematics education at the undergraduate level, note that they, as trained mathematicians, perceive "a huge and intimidating gulf" between their experiences in their own coursework as students and the work of asking their own questions as researchers. While art education invites authorship of questions early, in math students typically are asked to provide accurate answers to questions posed.

The expectation of rigorous experimentation, exploration, expression, and authorship increases throughout high school and undergraduate levels of art education, with the expectation of greater sophistication and an awareness of the relationship between content, format, medium, and context. These characteristics of students' art making are similar to those exercised by professional practitioners, and therefore, students may choose to be exposed to the knowledge-production processes of artists increasingly throughout their education. Meanwhile, most high school and early undergraduate mathematics courses focus on imparting increasingly complex tools, and while the mathematics taught becomes more complex and abstract over time, and at times it may invite experimentation or flexibility in the ways students approach a question, it is not until later in advanced undergraduate mathematics that students experience creative authorship in a way parallel to the arts. At the earlier levels of education, there is "an undue emphasis" on the skill-based applications of mathematics (Whitcombe, 1998).

The iterative acts of investigation, reflection, and revision become more complex as art students progress through their curriculum. As they ask more complex questions via their art and evaluate whether their artistic investigations are successful, the process requires that they perceive and respond to more complex emerging relationships and connections among ideas, potentially changing their method, their perspective on the question, or the question itself (Sjöholm, 2013; Walker, 2004). In mathematics education, up until the advanced undergraduate level, iterations of investigation, reflection, and revision are engaged primarily as they are needed to choose a mathematical tool from those already taught in order apply it to solve a given problem, and to reflect upon and revise the choice of tool or approach if the selected one isn't working. The iterative behaviors of the creative process are invited and expected more of students in art than students in math, primarily because, in art, they have authored the starting question or idea.

This difference in the invitation to engage in inquiry may potentially be explained by the idea that creating math requires an entire language with a specific grammar. When Halmos asks "why mathematics occupies such an isolated position in the intellectual firmament," he suggesting that the language is one reason, noting, "it takes years to learn to speak it well." Others argue that all people are capable of mathematical inquiry, at all levels, and that schooling should make available and empower all students to these experiences (Katz & Thoren, 2017). We teach the behavior of authorship with limited language skills in other disciplines, so why does education prioritize the language over authorship and inquiry behaviors in mathematics?

Dispositions

This difference in authorship also affects the experience of openness and perseverance in mathematics education and art education. In art education, the act of idea generation or question-forming requires openness and curiosity, and at more advanced levels, the exploration of the question though iterations of reflection and revision requires a tolerance for non-closure, an openness to ideas emerging during the process, and ongoing curiosity as the process unfolds. At higher levels of mathematics education, this openness is also engaged as the level of authorship increases, though in earlier experiences, an emphasis on accuracy discourages risk-taking and curiosity (Mann, 2006). Students are asked to exercise and develop perseverance in both disciplines, in perhaps somewhat different ways, as longer-term projects in art and challenging multi-step projects in mathematics both require durational effort and multiple attempts in the face of challenges. But mathematics education often requires perseverance towards a correct answer that is known by the teacher or book, while perseverance in art involves staying with a process when the conclusion is yet to be determined at all. The greater the freedom to choose or revise the question, and the more exploration of ideas is encouraged, the more the process is responsive and engages openness and perseverance. Essentially, when the taught processes resemble those of the professionals, the dispositions of the process are invited.

In an article reporting on their experiences and those of other colleagues who also mentor undergraduates in mathematical research, Dorff, Henrich, and Pudwell (2017) present a list of a dozen points faculty know but their students probably do not know, and would benefit from learning, about mathematics research. Of these points, the majority relates directly to dispositions of openness and perseverance: don’t be afraid to ask why; be open to different ideas and approaches; a project may go in a different direction; it’s ok to make mistakes; and perseverance is necessary but not sufficient for the process. Other pointers are about staying with an idea, coming back to an idea, and being patient with an idea that is taking longer than planned. This advice to faculty mentors suggests that the authors and their colleagues have found it important to teach the dispositions of the process as students begin engaging in mathematical inquiry and authorship.

Epistemologies and Aesthetics

Where do students become exposed to thinking like a mathematician or artist and to artistic and mathematical ways of knowing and making meaning? Because disciplinary canons, methods, and epistemologies are interrelated, ways of knowing are partially introduced as the canon and methods are presented throughout schooling. In addition, as art students move into more serious artistic study, artistic ways of knowing are demonstrated through collective conversations about artworks and are made yet more transparent through critiques, in which the choices of the work are rigorously contemplated, often in relationship to the traditions of the discipline. In mathematics, the disciplinary epistemology is embedded in the symbol system and grammar, so students are partially exposed to mathematical ways of knowing, like with art, via the partial introduction to the mathematical language they receive in K-12 schooling. But revisiting the quote by Halmos (1968), it takes years to learn to speak the language of mathematics well, and the epistemology of the discipline may be regarded as difficult to grasp with only a limited view of it.

The exposure to aesthetic values in these disciplines differs widely, resulting in another reason why art and math are viewed so differently, even when their processes are so similar. As art is seen as the exemplar of aesthetic values, aesthetic choices are intertwined with the curriculum at all levels. Student work is lauded for its aesthetic qualities, as are the historical examples students are presented. Furthermore, students are also often exposed to the aesthetic values of art from different cultural contexts, providing them a window to understand that art is socially-constructed, and its definition may vary across contexts.

While elegance is a quality of a well-answered question in mathematics at all levels, direct focus on the aesthetic values of mathematics varies widely until higher levels of mathematical engagement. Yet, as Mann (2006) notes, to deprive students an experience of mathematical beauty is to deprive them of exposure to mathematics. And Whitcombe (1998) describes UK and US mathematics education as an “impoverished curriculum” because “the wellsprings of mathematics are not utility and relevance, but creativity, imagination, an appreciation of the beauty of the subject,” and when students don’t learn the “difficult-beautiful-rewarding-creative view of mathematics,” they don’t engage with the essence of the discipline or its process. Taking his critique further, Whitcombe asserts that, “most children’s mathematical journeys are in vain because they never arrive anywhere, and what is perhaps worse is that they do not even enjoy the journey.”

While these processes are similar and involve parallel behaviors and dispositions, they rest on different epistemologies and occur in different symbol systems or mediums, and the traditions of schooling take different points of entry to the disciplines based on these disciplinary languages and tools, and based on the role the disciplines play in the culture. Looking at art as cultural and personal exploration invites students to exercise curiosity, idea generation, and exploration. But when math is viewed as valuable primarily for its application to finding solutions and achieving precision, algorithmic methods are used as a point of entry. With a shift in perspective, looking instead at the similarities in the disciplines themselves, an introduction to either discipline could start with curiosity motivating the generation and exploration of ideas. In some ways, the introduction to the creative process in art can be seen as an introduction to a good deal of the iterative, curiosity-driven, aesthetics-guided dispositions of the mathematical process.

Participating

In order to make a contribution to the artistic or mathematical discourse, a practitioner needs an understanding of the historical canon and the contemporary conversation. Beyond this complex canonical knowledge, a practitioner needs expertise with a selection of tools, and enough of an understanding about what each provides with regards to transforming information or meaningfully formatting content to select from amongst them for a given inquiry. And more than this acquired knowledge and skill, the practitioner also needs to *know how to learn* new information and tools as needed. Therefore, in both disciplines, the understanding needed to make a meaningful contribution extends beyond what traditions of typical schooling provide, until advanced higher education– or serious personal practice and self-teaching– prepares the artist or mathematician with enough understanding of the disciplinary conversation to join it meaningfully.

The audience for mathematicians' work is often other mathematicians, who serve as both the gatekeepers to the discipline and the consumers of it. The process of making new math is geared towards this audience. The audience for art is broader and includes other artists, the general public, and the curators, collectors, and critics who serve as the gatekeepers to the discipline, and to its often-related art market. Not all audiences of art are looking for the same thing. While some viewers may desire to be at the cutting edge of art canon creation and artistic definition, some consumers want investment pieces or appreciate objects that fit a particular aesthetic sensibility, and some people look to learn or have a particular, connected experience offered by art-viewing.

Furthermore, the entry point to math is an ability to read math; the entry point to art is an ability to *experience* art; art is more available for its broad address of multiple audiences and its point of entry. While they are both cultural products of socially-constructed fields, one discipline presents itself more readily to the culture, and therefore can "recruit" participants and engage interest more broadly, while the other recruits, trains, and presents its process and products primarily within advanced higher education, providing little exposure to those outside the group, which creates another hindrance to overcoming the inaccurate popular opinion of the discipline.

There are also more personal enrichment opportunities in art than math– exhibitions, arts classes, and arts events take place at art centers, public libraries, museums, galleries, and cultural festivals. Art is regarded as a "natural" part of culture, extending a universal invitation to create and experience, whereas math is often presented (from within and without) as separate from culture. This may be another contributor to the miscomprehension of mathematics in comparison to art: art is accepted as cultural and personal right, but who has a right to math? And how does this affect the perception of this discipline and the choice to engage in it long enough to arrive at the glorious moment when the traditions of advanced schooling or rigorous personal practice make creative contributions possible? As creative fields in an education and cultural landscape that lauds creative engagement, one might expect pure mathematics and fine art to extend equally inclusive invitations to participate, but it seems that there are obstacles to mathematics that obscure meaningful access to creative engagement.

That being said, the creative process of mathematics is available freely to those who pursue it. All one has to do to create a mathematical object is imagine it, with no materials list required (Wenzel, 2018). Mathematical puzzles and games have long been part of cultural activities, and there is also an active community of recreational mathematicians who engage in the mathematical process, much as there are recreational artists who engage in the artistic process, with fewer constraints of the canon and the contemporary disciplinary conversation–the end product of these pursuits is personal exploration, enjoyment, and discovery, rather than a discourse contribution or disciplinary recognition. To participate in recreational art or to have a personal art practice requires only basic skills and the desire to learn the techniques one wants to use; much can be self-taught. To engage in recreational math, one similarly needs only the desire to engage and a basic set of skills, available in typical schooling or through self-teaching. These recreational pursuits resemble the behaviors of artists and mathematicians in their potentially iterative behaviors of inquiry, investigation, reflection, and revision, and they exercise similar dispositions and affective experiences, making much of the creative process available, minus the concerns and limitations of relevance to the contemporary disciplinary conversation.

This all raises the question of purpose, both regarding the projects of contemporary math and contemporary art, and regarding education towards them.
As both pure mathematics and fine art are curiosity-driven pursuits, looking at the behaviors and dispositions these processes entail, one potential purpose of education and engagement in these disciplines is to rigorously engage in curiosity and wonder. To wonder indicates not knowing and an openness to investigation. It implies inquiry.

Another strong argument for educating toward the creative process in these disciplines is the behaviors themselves, which to some degree underscore not only these two disciplines, but all curiosity-driven creative processes to create new knowledge. An education in these iterative behaviors exercises openness and perseverance and fosters independent, reflective, and critical thought through multiple lenses. In this way, pursuing curiosity and wonder in mathematics and fine art can be seen as exercising the intellectual behaviors that may lead to personal actualization, as individuals encounter themselves in the dialogical process of making choices and reflecting. Dewey (1934/1980) describes this phenomenon in *Art as Experience*, in which the creative encounter transforms the self through the doing and undergoing of perception and investigation.

These behaviors of sensitive perception, open reflection, and rigorous investigation towards a self-transformation are also seen as key aims of liberal education (Budwig, 2018; Neem, 2019), and while these behaviors are often discussed with respect to the humanities, here it is clear that they are part of the creative knowledge-generation process in pure mathematics and fine art. A perceptive and intellectually courageous citizenry is required for the self-governing acts of identifying questions and pursuing truths around complex issues. An education that exercises attuned perception, a habit of inquiry, iterative reframings, openness to new ideas, and perseverance with the process of investigation plays an important role in individual and civic life. For this reason, an understanding of and education towards the creative knowledge-generation processes of art and math may provide the inquiry, reflection, and investigative behaviors necessary to contribute to, and co-author, culture.

As higher education changes, it is essential to maintain a prioritization on these curiosity-driven processes, which teach students not only about engaging in inquiry and manipulating ideas within their symbol systems and materials, but how to engage more broadly in idea generation, to wonder in abstract thought, and to transform, clarify, and describe those ideas. Much as we value art for what we experience and learn in making it and observing it, we also can value math; they offer similar spaces for aesthetic experience and intrinsic motivation to guide inquiry.

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Paper 2: Informally Learning Mathematical Inquiry and Aesthetics in the Social Field

Abstract

This study draws on semi-structured interviews with nine faculty members in the curiosity-driven discipline of pure mathematics to query how the creative knowledge-generation processes is imparted and developed, with attention to the dispositions of the process and disciplinary aesthetics and epistemologies. This paper begins with an overview of the mathematical process drawn from accounts of mathematicians and math educators, noting behaviors, dispositions, the role of aesthetics, and mathematicians' concerns about schooling. It then discusses themes that arose in the interviews, primarily the ways that question-finding is learned and practiced socially, noting the role of mentorship, peers, and the greater mathematical community; and the development of aesthetic values as they play into the learning and practice of pure mathematics. A discussion relates the social arena of learning and practice to issues of cultural and social capital per Bourdieu.

Keywords: mathematical process, aesthetics, cultural capital, social capital, mentorship, mathematical inquiry

Paper 2: Informally Learning Mathematical Inquiry and Aesthetics in the Social Field

This study investigates the way that mathematicians develop their processes throughout their education and impart that process to their students. This is part of a larger investigation that asks how the creative knowledge-generation processes in the curiosity-driven disciplines of pure mathematics, the physical sciences, and the fine arts are imparted and developed, with attention to the behaviors and dispositions of the process and to disciplinary aesthetics and epistemologies. To investigate this broad question, I conducted semi-structured interviews with 29 faculty members in pure mathematics, curiosity-driven physical sciences, and the fine arts. Using a subset of nine of these interviews, the present paper discusses elements of the creative processes and education of mathematicians that arose in the interviews. The key themes that emerged described the ways that question-finding is learned and practiced, and the ways mathematicians develop their aesthetic sensibilities, noting the role of mentorship, peers, and the greater mathematical community. These findings can be related to issues of social capital as they play into the learning, practice, and authorship of the socially-constructed discipline of pure mathematics.

Summary of Background Literatures

Behaviors and Dispositions of the Process

Mathematicians have presented many accounts of the processes by which they work, noting the ways their working process is a creative one. The behaviors of this process– idea generation and iterations of investigation, reflection, and revision– are detailed in O'Grady (2021). Most notably for the current investigation, the mathematical creative process includes idea generation and inquiry– the choice of a question that will frame the phenomena to which a practitioner will be deliberately open. Ervynck (1991) discusses mathematical creativity as relational and selective, and it involves a choice in what to define or prove and how to prove it. Silver (1997) notes the importance of posing mathematical problems in fostering a creative orientation for students. Katz and Thoren (2017), in discussing the opportunities presented through inquiry-based learning in mathematics, point out the difference between the way they work as mathematicians and the way they were originally exposed to mathematics as students, arguing for the inclusion of mathematical inquiry in the curriculum to fill the gap.

Much of the work of investigating a mathematical question is described as an experiment, which may involve the creation and use of heuristics or models that are then used to explore ideas (i.e., Sriraman, 2004; Wenzel, 2018). Wenzel notes that "underdetermination can be a good thing" in creating and transforming these models as it leaves the process open to new ideas. The investigative process is iterative and durational, as a mathematician "makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions… arranges and rearranges [their] ideas and becomes convinced of their truth long before [they] can write down a logical proof…" which can result in the awareness that the idea is incorrect, and the process begins again (Halmos, 1968, 178). In short, these mathematicians describe an iterative, non-linear process of inquiry, experimentation, and revision. This iterative process also requires and exercises openness and perseverance to engage in experimentation, knowing that it might yield unpredictable realizations or require a new approach (i.e., Mann, 2006; Wenzel, 2018). Halmos' description above illustrates the perseverance of the process as a mathematician might work for months, making many attempts, before ascertaining whether an idea is correct, potentially starting over. Ervynck (1991) and Mann (2006) depict a state of mind prepared to forge new connections about ideas; Mann asserts that this state of mind can be fostered in education.

Aesthetics

The idea that the mathematical process is a creative one often arrives connected the idea that it is guided by aesthetics (El-Sahili et al., 2015; Dienes, 2004). Sinclair (2004) determines that aesthetics play three roles in the processes of mathematicians–motivational, evaluative, and generative–which would then guide the choices of the process. In interviews reported by Karakok et al. (2015), mathematicians discussed their preference for engaging in an aesthetic way of working. Whitcombe (1998) asserts, "The wellsprings of mathematics are not utility and relevance but creativity, imagination, and an appreciation of the beauty of the subject," suggesting that we need to teach children to speculate and look for aesthetic qualities in mathematics. A number of mathematicians have directly compared their processes to that of artists, often grounding their comparison in the fact both processes are concerned with an aesthetic sense (Wenzel, 2018; Halmos, 1968). Hardy (1940) states, "There is no permanent place in the word for ugly mathematics." Putting these accounts together, aesthetic motivators affect perseverance and openness in engaging in a nonlinear process of inquiry and experimentation.

Concerns about Schooling

Mathematicians' concerns about how mathematics is taught often point to a lack of attention to math's aesthetic qualities and inquiry behaviors. Famously, Lockhart (2002) lamented about the conventions of mathematics education for these reasons, describing them as boring, stupid, and a nightmare. Mann (2006) explains that teaching without attention to the aesthetics of math deprives students of true exposure to the subject, noting that this is common. And Silver (1997) argues that math education must attend to problem-posing along with problem-solving in order to avail of the mathematical process. These criticisms address a fundamental difference between the way math often appears in early schooling and the way it is practiced by mathematicians. But clearly, mathematicians emerge from their early education, regardless of the inconsistencies it may present in addressing the full creative process of mathematics, prepared to choose and engage in advanced mathematical study and, from there, to author new mathematics. This study then asks how mathematicians develop their processes and how they impart the creative mathematical process to their students. Attention is paid to the way mathematical inquiry and aesthetics are recognized in the process, developed, and shared.

Social Field

Many accounts of mathematical practice seem to depict a solitary experience (i.e., Hardy, 1940), but the interviews here reveal a social narrative in the learning and practice of math that contextualizes the ways that the mathematical process is imparted and relates to the socially-constructed discipline of mathematics. This relates the mathematical process to Bourdieu's (1986) conceptualization of social capital. Sriraman (2004) reports on interviews with five artists, finding that social interactions such as conferences and emails with colleagues were important to their processes; they also valued the interactions they had with their graduate students. This research is attentive to the role of the social and solitary environments of learning, whether informal or formal.

Methodology

This present study is a part of a larger investigation with faculty participants in curiosity-driven disciplines in the arts, physical sciences, and pure mathematics to query how they practice, learned, and impart the creative processes of their disciplines. This paper is only concerned with the mathematicians.

Overview

To investigate how mathematicians develop and impart their creative research processes, I conducted semi-structured interviews with nine mathematics faculty members who are engaged in both teaching and research. The semi-structured interview format invited participants to share their experiences around a few broad questions and also created space for unexpected ideas to arise and to be clarified with follow-up questions that drew on details provided from within the participant’s narrative (Bogdan & Biklen, 2006). This allowed the interviews to focus on, and further query, the teaching, education, and research experiences the participants described, which is in line with the interpretivist paradigmatic stance of this research, seeking to understand the meanings of the participants’ accounts through rich, personalized narratives (Lather, 2006; Schwandt, 2000; Pascale, 2011). Interview data were analyzed to both look for findings addressing the primary, exploratory research questions and to attend to ideas emerging in the data.

Participants

In order to address the ways that creative processes are imparted and developed in higher education, the participants for this study were faculty members in pure mathematics who were found through searching faculty lists at R-1 universities, though two of the interviewed faculty taught primarily at liberal arts colleges. The faculty were actively engaged in research and teaching as confirmed by faculty websites and course listings. As this study focused on curiosity-driven disciplines, it did not include faculty from applied mathematics. Participants were from four U.S. colleges and universities.

Sampling was purposive to gather the perspectives of mathematicians from various sub-disciplines, era of graduate study, years teaching, gender, and race/ethnicity, with attention to the fact that these differences may have had an impact on participants' experiences of schooling, teaching, or academic environments. With this in mind, I selected mathematicians by reading faculty profiles.

The participants were at different moments in their career and ranged from those very well known in the discipline to newer faculty who were just getting established. They had pursued their educations and entered teaching at different times, which was important given the changes that have transpired in the past few decades in higher education, and which may have affected their experiences as learners or teachers. Participants had received their PhDs in the 1970s through 2017.

As this investigation was particularly interested in how successful researchers choose to impart their processes to others, an effort was made to include participants who had received teaching awards; two of the mathematicians had such awards listed on their websites.

Noticing that many faculty members were educated abroad, and considering the potential ways such diversity may enrich or complicate the data regarding education and teaching, I deliberately contacted several internationally-educated mathematicians but maintained a critical mass of U.S.-educated faculty. Two of the nine mathematicians in this study were educated abroad, and one was educated abroad until college.

In reading faculty web pages, I noticed that, of the women present, many were educated abroad, and many were in applied math or practiced applied sub-disciplines. The U.S.-educated women in pure mathematics were even fewer, whereas the U.S.-educated men in pure mathematics were the overwhelming majority. In light of the gender disproportionality in math (for example, see Natanson, 2017; Hu, 2016; Glazer, 2019), I attempted to include 50% women or non-binary individuals. In order to achieve this goal, I contacted women at more than double the rate of men, and at a greater number of universities.

While an effort was made to recruit participants from a diversity of racial backgrounds, there were so few faculty from underrepresented minorities on the websites of the departments considered that none of the faculty interviewed were from underrepresented racial backgrounds; this lack of diversity will be addressed again in the discussion.

Recruitment

I emailed selected faculty members at their university email addresses and provided a brief description of the project, asking whether they would be interested in participating. No financial incentives were provided. To pursue recruitment goals, faculty were contacted in sets, awaiting responses before contacting additional faculty, responsive to the evolving list of participants and the topics and backgrounds they brought to the research.

Overall, I emailed 25 mathematicians and conducted nine interviews. The response rate was 36 percent, with differences by gender. The response rate was 25 percent for women and 55 percent for men. The lower response rate for women was possibly due to the frequent asks women in the discipline receive; one participant mentioned this in the interview.

Interview Protocol

The approved interview questions addressed the following main topics: How the faculty members learned how to engage in their current creative/research process; their description of their process; and their goals in teaching their students with respect to this process. I paid particular attention to whether the descriptions they offered addressed idea generation or experimental processes, mentorship or advisorship, autodidaction, socio-emotional dispositions (often rephrased in the interviews as "attitudes to the work"), aesthetics, and social environments. If these topics did not come up in the course of the conversation, I asked follow-up or protocol questions on these topics.

After several interviews, it became clear that the protocol was too long. Extending the notion of emergent design (Jacob & Furgerson, 2012), which I was employing to choose relevant protocol or follow-up questions to ask within the interviews, I chose to revise the protocol in a more deliberate way by honing in on the most pertinent questions. I reviewed the early data, paying attention to which questions were arising naturally and which were yielding rich descriptions, and I reprioritized the protocol, leaving many questions off the list during subsequent interviews.

After an early review of half of the data from the full study (including artists and physical scientists), it became clear that informal learning and social environments were a significant part of the participants' experience, and I chose to add a broad question regarding interviewees' experiences with bias and/or privilege in their learning, teaching, or research environments. The last third of the interviews with mathematicians included this question, and I contacted most of the previous participants to invite them to a follow-up conversation on this topic (some had already addressed these issues in the course of the first interview and were not contacted for a follow-up.) Two participants agreed to a second conversation.

The interviews themselves were semi-structured. Each interview addressed the main questions of the protocol, while the conversations were allowed to flow organically within each question. In some cases, I asked a number of secondary and tertiary questions from the protocol, and in other cases, I asked only the main questions and direct follow-up questions based on the participant's responses. Participants naturally had different conversational styles, and some responded more to a series of shorter, discrete questions, while others spoke more narratively in response to broad questions. For the purposes of this exploratory research, this variation in conversational style allowed for the participants' to provide context where they wished, while allowing for follow-up questions to deliberately pursue subsequent questions at other times. One limitation of this approach was that, due to the time-limited nature of these interviews, some topics were not covered in all conversations. An emphasis was placed, instead, on thick description and on follow-up questions to pursue tangible and immersive detail, such that I could picture what they were describing. This is related to what Tracy (2010) refers to as crystallization, seen as an alternative to triangulation in pursing clarity that addresses subjectivity.

Interviews were conducted in person, over video call, and over the phone. They lasted between 45 and 90 minutes. Prior to the interview, I reviewed the consent form with the participant. For video and phone calls, I asked for verbal consent including three yes/no questions that addressed recording and transcription. For in-person interviews, I provided a paper consent form, which we both signed. When consent was given, I audio-recorded the interviews; I made an exception for some phone calls, preferring to take notes rather than record, as recording the call decreased the audio quality during the conversation.

Analytical Strategy

Analysis began in the interview. Because of the abstract nature of the constructs queried and the relationships between them, it was often necessary to present my interpretation of what the interviewee was saying, as either a summarization of points they had made to contextualize another question, or to present an interpretation, asking them to correct it. These in-process analytical moments both introduced and removed bias from the interviews by making evident that my interpretation and priorities were guiding the conversation, particularly in follow-up questions, and by deliberately inviting participant checks on some of those interpretations. These moments also increased the conversational tone of the interviews.

I transcribed some of the interview audio files directly. I cut other files for anonymization and relevance, sent them for transcription to a company that had provided a non-disclosure agreement, and then validated the transcript. Both of these processes served as a way to become familiar with the data, which was furthered by iteratively reviewing the resulting transcripts, with sections highlighted, categorized, and annotated in comments; these annotated transcripts served as working documents for emerging ideas, and the categories eventually became codes.

I organized data by main topic in a spreadsheet early in the process. This spreadsheet allowed for a quick glance at which questions were yielding rich data. This worked particularly well because the intended topics of the interview questions did not arise in tight, clean units; they were often alluded to across multiple different answers. The spreadsheet allowed me to visualize the topics covered across all interviews. I often summarized the data in prose, sometimes including quotes. The resulting spreadsheet summarized data by topic code (per Saldaña, 2013) and participant.

Once the interviews and dataset were nearly complete, I began to code the data using both Atlas TI and Quirkos. I realized part of the way into this endeavor, though, that by iteratively reading the files and organizing the data into the spreadsheet, I had topic-coded all the data and already arrived at the cross-topic themes of this paper. Realizing I was duplicating my coding efforts to justify results already attained, I instead discontinued two practices: rereading the transcripts to prepare them for coding, and coding in a software program. Instead, I read the remaining transcripts and entered them into the spreadsheet of data, updating the spreadsheet with a more elaborate, detailed set of topic codes, and adding the themes that were emerging. I also reviewed the data from the previous participants to distribute data into the more detailed codes.

Iterative writing served as a final and important part of analysis. After summarizing data into the spreadsheet, I began writing a memo on each participant with regards to the main themes. These memos were then reviewed, compared, and annotated, yielding an outline of interrelated themes. I continued to revisit the transcripts and audio files during the writing process.

Findings

Due to the semi-structured and exploratory nature of the interviews, data were heterogeneous as they addressed the unique experiences of the individual faculty across moments in their careers. Within these varied accounts, several consistent themes emerged, and those themes were interrelated: The challenges in finding the question, the ways it is found socially, the ramp-up to learning how to find this question, the development of a personal mathematical aesthetic sense, and the role of cultural capital and social capital (as discussed by Bourdieu, 1986). These themes are discussed below, presenting parts of the narratives from which they emerged.

Theme 1: The Question is Often Found Socially

Seven of the nine mathematicians in this study discussed the challenge of finding the question to work on, and of the nine interviewed, four discussed how this is the hardest part of their process. Notably, this contribution was often offered in response to a question about how they found their ideas; there was no protocol question asking for the most difficult part of their process, so it is possible that more of them share this perspective. As Math One says:

> … new PhDs for example, postdocs, have a lot of trouble finding their own problems to work on … a very hard part of the process of becoming a mature kind of researcher is finding good things to work on. Math is particularly hard because it's so large an enterprise and so old an enterprise that the literature is vast.

This part of the process is rarely taught before graduate school, and at that, many students really do not experience the challenges of this question-finding task until after they have received their PhD. Math Two recalls advice from his advisor: “It was the norm in research, you look for something and then you find something else, so you follow your way, it’s funny, by luck somehow.“

Generally, the mathematicians mention three ways of finding their questions: Sometimes a new question will arise naturally from something they are working on or have already done. Sometimes they can find an open question in the literature that speaks to their interests and skills, or they can look to generalize or apply other mathematician’s findings to certain examples. But the way that the majority of the mathematicians emphasized is social: They find the question through interactions with other mathematicians. Two of the mathematicians used the phrase “happenstance” in describing how the question can emerge socially. And the narratives offered in the interviews highlight a deliberate choice to be present, physically (or virtually) and intellectually, to the ideas of others in order to engage in considerations that might yield a new question.

Math Five offers emphatically that finding the question to work on is the hardest part of being a mathematician. He describes that community and collaboration are very important for him in finding his next question: “… just networking and communicating, going to conferences … is actually more important than just trying to read a million papers. It’s just interacting with people.” He advises his students to go to talks and interact with others, admitting that a talk can be frustrating but still might yield an idea that you can use. “These kinds of things can always happen, and you can’t be isolated.” Because questions arise socially, he explains, “That’s why you want to have a good math department with other colleagues you can talk to.” Math Four agrees that ideas are often found in the social spaces of the field, and adds that when starting out, you are also learning how to talk to people in order to take advantage of these social spaces.

Math Seven, an early career mathematician, agrees that, “really, the hard part is finding the problem … that is … interesting, has never been proven, and is doable, basically.” She describes her PhD advisor as an expert in this, telling how he advised her to read recent papers looking for open questions, and describing his familiarity with the literature as a reason for his success. Quickly she clarifies:

> I think it’s more about his connections with people. He has so many former students, so many colleagues, that he has people telling him all the time… he talks to people and they’ll ask him a question, and he’ll be like, “oh I don’t know the answer to this, but maybe this is a good problem for a grad student” … or he’ll work on it himself.

In this way, his social network contributes questions for himself and his students, which she describes, “It’s a self-perpetuating thing, because he’s already such an expert in the field, and so a lot of people come to him with questions.”

Math two describes a recent conference where he was interacting with participants from multiple sub-disciplines of mathematics “to get new ideas, to be aware of what is developed elsewhere.” He explains, “… it's not developing mathematics with your eyes really downward with … [a] really narrow vision. It's time to stay open to many possibilities and see what happens.” He discusses the importance of “listening to the math," in his community, describing a more senior colleague who has "this ability to really hear the mathematics and … see which view will be best suited for the purpose." He explains:

> … it's a very hard sense to get. It's like if you want to practice music, you have to train your ear. And it's really rare when you start music to have a perfect ear at first … The kind of community I'm in… is really about trying to get good notions … and for that you have to develop your listening skills somehow…

Math Six adds that one mathematician will approach another at a conference with a question about a topic they address from different directions, which can bring about new questions. Illustrating how two people can differently approach a topic, she recounts an example where a mathematician with whom she was exchanging ideas did not recognize his formula because she had converted it to a rule (in code), "… but since my perspective is more looking into patterns, I found the pattern … he wasn't able to see, so that is one way that you can look for problems."

These descriptions of finding questions socially all had a texture of shared wonder. Even where the research agendas are quite ambitious and address longstanding problems, the mathematicians describe socializing and questioning based on shared curiosity about abstract ideas. Bringing this together, Math Nine teaches her undergraduate students to ask "wondering questions… I wonder if we change the hypothesis, I wonder if we use this example instead," telling them, "that's how new mathematics is developed." And in describing his teaching, Math Eight says:

> I think it's important to realize that it's important to ask the right questions– that questions are sort of more important than the answers, and the process of getting the answers is more important than the answers themselves… try and ask why...

Even though asking the question is a challenge, these mathematicians demonstrate that undergraduate mathematics courses can introduce the behavior and texture of mathematical inquiry.

Theme 2: The Role of the Advisor in Finding the (Usually Solvable) Question

For most of the mathematicians interviewed, their PhD advisors gave them their dissertation question or provided a topic or a set of options with some degree of flexibility, because the graduate students generally aren't prepared to choose a question of their own. For instance, Math Nine recalls that she and her advisor "had been reading something together and I kind of told him what I liked," and that he then "pretty much gave me a topic." Recognizing what worked for them, and now as advisors to PhD candidates, five of the mathematicians discussed how they provide the question or a few specific options to their students, based on their emerging interests. And Math Four describes how her advisor helped to formulate the questions she was going to ask in her research but then let her go her own way to struggle with it, stepping in periodically to offer suggestions.

Math Two describes that there are two kinds of PhD advisors: the kind that provide the question, and the kind that don't. Unlike others interviewed, Math Two found his own question, and he did so in the social spaces of math, from a series of lectures in his city that addressed his interests. While his training involved a good deal of independent proving early on, so perhaps he was well prepared for the task, he says, "it was a shock to find … your own question," but he describes how it was a really helpful and formative experience, "because at the end of the PhD, you have to be able to be alone in your research, so it's good that you have your own questions already."

Typically, advisors provide their graduate students questions that are vetted to be solvable, but not always. One mathematician, finding himself well prepared for graduate school based on many additional courses (beyond the major requirements) during his undergraduate studies and a summer of self-teaching, was half-joking with a young faculty member, sharing that he didn't know what to do with his time. The faculty member gave him a couple of famous papers to read, and the interviewee recounts how he then talked to his soon-to-be advisor about the papers, who quizzed him on the board and then agreed to work with him, giving him an open question based on those papers. His advisor warned him, though, that he didn't know whether the open question was solvable; however, in this instance, he was able to solve it. This was a fortunate exception to the rule, as the faculty here mostly described situations in which the advisor creates opportunities to provide the student a process with more confidence that they will succeed.

For most mathematicians then, the task of independently evaluating whether a question is solvable and "good" is left for after the PhD, which can present them with a challenge. Math One describes it generally: "You do a PhD, and pretty much your advisor sets you on some path and problems to work on, and then you work on that, but then maybe you solve it, or maybe it's completely solved with other people, and then what do you do? You have to find something to work on, and it's not so easy." As an example of this, Math Four's advisor had guided her in finding her dissertation question, but when she was doing her postdoctoral fellowship, she describes really struggling personally, for three years, while trying to find new questions, even as a self-described optimist. It was when she found a new question to work on, after solving something during her postdoc, that she finally felt like a real mathematician; the successful question-finding affected this self-regard.

For most of these participants, their experiences line up with Math Two's first category: the advisor chooses the advisee's question (or guides the choice very significantly). Put in dialogue with the participants' frequent assertion that questions are found in the social spaces of mathematics, this may indicate that the important work of learning how to find one's questions socially is often left to informal learning and modeling, to be tried to a later time. Math Seven, who is still working with questions that follow from her dissertation ideas, says that she does not yet pursue her questions in the social arena like her advisor did, though if she "worked harder on that, then [she] could cultivate those connections a little bit more."

Theme 3: Learning Aesthetics in the Process

Unsurprisingly, even when unprompted, all of the mathematicians in this study discussed the aesthetic qualities of good mathematics and the ways those qualities are part of learning the process. This resonates with the abundant mentions of mathematical aesthetics in the literature. The mathematicians used terms like elegant, cool, beautiful, pleasant, or pretty to describe mathematical questions, proofs, tools, and their own experiences of working with and refining abstract ideas. They made comparisons to art, music, and poetry. Aesthetic values underscored their mentions of affect and motivation, and these shared values seemed to bring communities together. Also, several mathematicians described the evolution of an aesthetic sense as playing into the important and difficult task of finding a good question to work on.

Math Four explains that a good mathematical question has something pretty to it, and in response to a question about how she learned which questions were the pretty ones, she says, "you sort of develop a sense. … any kind of sort of aesthetic, you sort of have to develop it over time." Math Three discusses how, by learning mathematical tools, students develop an appreciation for elegant or beautiful questions, explaining, "…you can't sense a good question until you have tools … It's sort of like not being able to read poetry until you understand the language… I can hand you the world's prettiest poetry in Arabic … and it would mean garbage to you until you understand Arabic." He details a long process of arriving at an understanding of the language and its elegance simultaneously, speaking generally and about his experience as a student and advisor:

> …you learn the tools, and you see how those tools solve these questions. And in doing so … an interesting thing happens … where you learn … what is elegant and what is an interesting question. There's no particular person who will tell you, "this is a nice question, or this is an elegant solution," but it sort of happens naturally … an acquired taste, but it's a consistent acquired taste.

Extending their explanations about why advisors choose questions for their students, both Math Four and Math Three also discussed the role of the advisor in the choosing or significantly guiding the choice of an *aesthetic* question for a students' dissertation work. And given the organic way a mathematical aesthetic awareness then might evolve over a long period of time, students are likely develop this sense by exposure to aesthetic questions and solutions provided to them.

Theme 4: Developing a Personal Aesthetic Sense

Another part of the education of a mathematician involves finding an area or sub-discipline within math that you like and where you are likely to have insights, which are related. Math Four always liked "shapey" kinds of things, landing in a branch of geometry, saying that, "I always thought in these sort of bizarre images. And that's why I think it's important that people pick the right sort of branch of mathematics, because I think you have to play to your strengths…" Math One concurs, saying that in addition to learning the material, "they're also individuals … they have their own style ... what you have to learn as an individual is to find your strengths, and where your creativity comes from." Math Eight found his area of math during a course in his last year of undergrad, after starting to feel like the types of more formal mathematical abstraction he had previously been studying and enjoying were "somewhat dry" in contrast to the new kind of math he encountered, which avail of a process he enjoys, walking around and letting ideas come together in a "wishy-washy visualization."

During a summer undergraduate research experience, Math Nine met a student from a different school who would talk about "developing your mathematical taste function" and who would describe certain theorems as "trivial" or "important". Math Nine was surprised to realize that she was allowed to, and was supposed to, have her own opinions about these ideas in math, even as an undergraduate. She compares this to finding your personal taste in music, saying that it's "weird" to say that you like *all* music, and similarly with math, it's important to learn what you like. And in graduate school, she was told that her classes were not about mastery, but about finding what was out there and what you liked, knowing you could go back and find it.

With altogether less strategic assistance from a program or mentors, Math Six found an area of math that appealed to her. She had chosen courses in her undergraduate education to avoid certain popular types of math that she did not enjoy, and then she taught herself a new area of math for her PhD work, since her assigned question differed from her expectations based on her graduate school application process; in her country, one applies directly to a research project at a university. Fortunately, through all of that, she says, “I ended up doing something that I love. Like I fell in love with it. It was like, this is great.” And because she socialized with members of the department who worked on the project she had expected to join, she could compare what she was doing to what they were doing, saying that she preferred what she found, as the other project “seemed really complicated.”

Math Three takes a deliberate approach to putting students on the path to finding an area of math that appeals to them, training his PhD students to be generalists rather than specialists and providing experiences with a wide variety of mathematics beyond what he studies, so they can determine what they wish to do later. And Math Four says she gives her advisees lots of things to think about with the intention that they form their own tastes and begin to consider what they want to do, but she also says she “basically give[s] them some very specific choices” for a dissertation project, because “…it's an exception if you have a student where they can say, I'm really interested in this. I want to solve this problem. That's rare because really you need to have a little bit more experience.”

This all amounts to more than finding something you enjoy. It also has to do with which topic's elegance you can best understand and perceive. To continue Math One's quote above:

> … math is very intuitive, and you have to kind of be insightful … So that's a long process of finding what it is you're really good at … and the kind of … insights that you have…. Because there's so many different people, and brilliant people, and they're all different. They think in a different way, they work in a different way.

His point about the diversity of approaches and topics, and the need to find an area where you can have insights, comes back to a fit between the mathematician's aesthetic sensibilities and the aesthetics of a particular kind of math; a good fit between individual sensibilities and a branch of mathematics then more readily avails of insights to open questions. Therefore, the selection of a resonant sub-discipline within math is an important part of setting up a student to enter a community and to address questions they are likely to be sensitive to, aesthetically, and to which they would have a better chance of contributing, insightfully.

Theme 5: Learning Aesthetic Values from the Field

The aesthetic values of mathematics take time, and exposure to develop, and the majority of the mathematicians indicated that they had at least partially developed their aesthetic values through reading papers, describing a sort of an incremental awareness of what constitutes good or pleasing math from what they read. Math Four says she developed her sense of good math from reading proofs, explaining, "I think you have to read other people's proofs … look at the construction and the rigor, and it's not obvious at all." But in response to a question about what she took away from reading proofs that helped her to learn what a good proof is, she admits, "I don't really actually understand that process that well." This slowly accumulating aesthetic awareness can be seen as a form of mathematical cultural capital, a set of mathematical cultural knowledge that is important for participating in the cultural field of math (i.e., Bourdieu, 1986).

Math Five experienced beautiful or cool mathematics from reading papers, remembering specifically those two aforementioned papers the faculty member gave him in his first year of graduate school: "They're definitely beautiful … just like out of the blue." He also switches between the terms aesthetic and important a few times, potentially indicating how much aesthetic values underscore what he considers to be important math. And like his mentors did for him, Math Three provides papers for his students, with the idea that in those papers they will be exposed to "an elegant solution or beautiful result," and, ideally, they will develop a sense of mathematical aesthetics. Similarly, when Math Six is working with independent study students, she will read a paper with them so they can discuss together whether it is a good paper and whether it is good math. The presentation of mathematical elegance is often curated, then, by an insider–the faculty member–as the student grasps the qualities of goodness present in the work.

Two mathematicians described a narrative aesthetic in the ways they read, present, and teach math. Math Seven describes how most math needs "a little bit more" to make it "good math," offering that what she does is "tell a story" with her talks and papers, in which describes a history that gives rise to an organic question that she then answers. She says that 95 percent of math talks are bad, due to the communication style more than the content, and she came up with her aesthetics of a good story because she wouldn't give a talk without understanding the point of it. "You have to be able to tell the story in order to show that your math is good math ... here's the story, here's why my math is good, and look at all these things that come from what I did." Another mathematician, who found proving to come naturally when he taught it to himself, now teaches his students with the story in mind, saying, "It's about telling a story … you have to create a narrative and … it does have to be correct in a … certain sense. But if you're reading a proof, you have to read it as a narrative and understand it as a narrative."

Theme 6: Aesthetics and Affect

A number of mathematicians discussed enjoyment of an *aesthetic experience within* the process, describing an affect arising while working with vague ideas as they clarify. Math Eight describes this affective experience: "For me, there's something when sort of things start to come together, sort of a very, very pleasant feeling of mist is lifting and you get a view of the whole." And Math Two, discussing a metaphor of a famous mathematician, says he prefers the mathematics that is "building the house" rather than "living in the house." He calls this type of process, which involves building new theory around a concept, as "really more creative, more artistic in some sense. And I prefer this kind of process." And for Math One, "… what's interesting is in the process …is to kind of get into a kind of dreamy state … it's intuitive, it's nonverbal … and then try to give it some life." Sinclair (2004) points out the motivational role played by mathematical aesthetics, and these affective experiences influenced by mathematical aesthetic sensibilities motivate the mathematician's engagement in what can be durational and challenging process. Math Three describes how aesthetic values can serve as a filter for recognizing solutions, as things become clear and fit together "like Legos," and Math Eight adds that things come together like a puzzle.

This affect also motivates engagement in the face of challenges. Most of the mathematicians described getting frustrated, many noting that it takes many attempts before you have results, and most days are spent without success. They keep going because "it's fun" and they love what they do. Math Two describes how, when his dissertation fell apart late into his PhD program, he kept going for the love of the ideas and "those kind of artistic things." He adds, "You really have to care about what you do." Math Four adds that even though most days yield no results, she is optimistic and stays motivated because "it's fantastic… I love what I do." Math One also describes how you have to enjoy the process to get through difficulties, comparing mathematicians to artists, who can work for hours and get lost in the process. Given how six of the mathematicians mentioned frequent frustrations, and given how aesthetics are seen to drive affect, an aesthetic fit seems important to persevere with the challenges of math. It is notable that the aesthetic qualities and experiences the mathematicians discussed are often made available later in education, after many students discontinue with the discipline, unless the student has the privilege of other opportunities to engage in math or is motivated early to teach themselves.

Theme 7: Building Social Capital and Networks in Graduate School

The mathematicians' early social contacts, both peers and faculty, provided them collaborators, sounding boards, and social capital. Given that important parts of the mathematical process are learned and practiced socially, including question-finding, the establishment of these networks is an important professional resource for engaging in the conversations and processes of mathematics.

Early peer networks as collaborators and knowledge resources

Early social networks, gathered around similar affects and motivations to engage in math, contributed to the development of the mathematicians' processes by providing shared knowledge resources and potential collaborators. For example, Math Seven describes how she sought and continues to seek help from her graduate school peers, particularly her friends, and says that she "definitely exploits her social connections" to conduct research, adding she thinks that is "the only way anyone gets any research done." Math Seven and Math Three recount drawing on peers for clarifying conversations about math topics rather than going to their faculty, because it could be intimidating to approach esteemed mathematicians with their questions. Math One recalls his first collaboration with graduate student peers who worked together intensively on a large chalkboard, describing the rapport graduate students develop by spending so much time together.

Learning from not having a peer network

On the other hand, looking back at his own education, Math Eight describes a relatively solitary experience, working in an area of math that was less popular at his institution and under an advisor who had no other advisees. His advisor did set Math Eight up with one of his collaborator's advisees at another university, who worked in the same area, and even though he enjoys working independently, Math Eight said it was really nice to have someone to talk to. Still, although he shared space in a graduate office, his graduate experience was a continuation of an independent undergraduate experience, based partially on a self-directed and competitive work style. Noting that he hadn't had a more involved peer network in graduate school, he offers, "that was one thing, in looking back, I felt like I did miss out on, because you do learn a lot from your peers." Nearly everything he does now is in some way collaborative.

Finding your own mentors

Math six pursued her social capital independently and deliberately in graduate school. Educated abroad in a system where graduate students apply to a PhD to work on a particular research project, she arrived at her graduate program to discover that her project and advisors had been changed, and then she found herself working very independently while her new advisors were rarely around. In addition, she had been assigned a topic that was new to her and fairly unique in the department. While none of the more-present members of her department worked in the same area of mathematics, she describes her eager participation in the department social life, which "created an environment for [her] to continue" in the midst of an otherwise isolated research experience. Eventually, though, she chose to study abroad to build her mathematical network, joining up with other faculties for a few shorter stays. The relationships she forged there led to collaborations, new ways of working, models of good teaching, mentorship that she continues to draw on, and the offer of a postdoctoral fellowship. Her deliberate choice to work with others with similar research interests at different schools brought her into the social spaces of her sub-discipline and opened up opportunities.

Early social capital: advisors

Math nine had the "very privileged position" of working with four extremely well-respected mentors throughout undergrad, graduate school, a postdoctoral fellowship, and her early career; she jokes that she is the most “over-mentored” person. She illustrates her PhD advisor’s influence, who encouraged her to apply to present her work at an event slightly after the due date, and when she did not hear back right away, he contacted the organizers, affecting an invitation to present within the hour. Math Nine’s undergraduate mentor also opened a gate for her by recommending an area of math to study in graduate school (which she had not yet encountered), recommending her future PhD advisor, and later, her PhD advisor recommended she approach her soon-to-be postdoctoral advisor. Social capital here created a direct path for opportunities to build yet more capital, though she points out that she had to be the one to follow through with the contacts and the work. She also did not realize how unique her experience had been until she was a faculty member working with early-career colleagues, realizing that they had not been told what her highly accomplished mentors had shared with her, and she adds that “people know who I am connected to,” as a reason she felt confident that she would be treated fairly as a woman in math.

Advisor’s social capital faculty created larger peer network

These early networks can last a whole career and can connect people across generations. Math Four describes how her PhD advisor had a great deal of success in mentoring graduate students who went on in academia, and generations of his advisees knew each other and formed a community even though they had not been at the university at the same time. Math One and Math Five talked about how they have often collaborated with their once-advisees after graduation, describing these communities as “mathematical families.” Math One explains that recent PhD graduates are not yet fully independent and want to keep working with their advisor, noting “this lasts a long time.”

And even for these more experienced mathematicians, they mentioned social networks that went back to their graduate school experiences.

Theme 8: The Privilege of Prior Education

The early education of these mathematicians afforded them an introduction to mathematical cultural capital, through exposure to its tools, logic, and aesthetics, which could be then used to build social capital in their graduate school and early research circles (i.e., Bourdieu, 1986). Math One describes the advantages of having a good educational preparation coming into college or graduate school, but in perhaps a different way than is commonly understood, and in a more salient way given the role of an advisor's social capital in a student's preparation to participate in the field. He notes that a good background in mathematics not only exposes you to more of the tools and content of math, but also affords the ability to knowledgeably choose an advisor with whom you want to work in graduate school, which sets you up to select and receive good mentorship, and to get teaching and research opportunities. Math One had gone to "great schools" that provided him early contact with "real" mathematicians, which he recalls as very important for his development as a mathematician. And he describes that he had this type of educational privilege in light of having few financial resources: "We were financially poor, but not poor in education."

Much as Math One describes, another mathematician knowingly recounts the privilege of a good background, describing a home environment with advanced math books and academic parents who pushed him along, one of whom had studied mathematics. He recalls reading these books as a child, figuring things out on his own, and having access to special programs. When he went to enroll in college, he not only placed into the accelerated honors track, but he insisted that he be permitted to skip the first year of it, teaching himself what was essentially the transitional course of analysis (proof writing) over the summer with the resources he had at home. Both through accelerated and additional coursework, and through well-resourced autodidaction, he prepared himself to begin his college career with a course “that was taught more at the level of a graduate course,” and when he made the choice of a less-popular mathematical sub-discipline in his graduate program, and therefore a much smaller and less connected graduate community, he engaged in his already-established independent work style.

The privilege of a good educational background also sometimes plays a role the choice of a sub-discipline that fits a mathematician’s taste. For instance, from his undergraduate coursework and self-teaching, Math Five knew what kind of math he enjoyed, and he knew where to go to study it for graduate school, and the mathematician quoted above found a topic in his last year of undergrad, after an accelerated start. These mathematicians had already been exposed to enough math to know their tastes prior to choosing their graduate school contacts, so the mathematical knowledge they accumulated early helped them find the communities of practice in which their mathematical taste resonated.

Discussion

The mathematicians in this study described the ways they developed, and taught others to engage in, the mathematical process. They describe the challenges of learning to find the question and of finding an area of math that resonates with their tastes and where they are likely to have insights. They also indicated the importance of the social arena for engaging in the mathematical process, including question-finding, and the importance of developing aesthetic sensibilities necessary to contribute to the field of math. And they describe how PhD advisors, peers, and other graduate school mentors guided them through the process.

Building on Sinclair's (2004) tripartite role of mathematical aesthetics, here we can see that in addition to playing motivating, generating, and evaluating roles, mathematical aesthetics *facilitate the social connections* that are important to mathematician's inquiry processes. Sharing in a common sense of what is good, beautiful, or cool brings people together to recognize new questions and to find collaborators. The mathematicians here were attracted to a mathematical aesthetic in order to choose the discipline, further developed that aesthetic sense through study, were motivated by it, used it to understand what good math looks like, and found themselves in the company of others, who were also attracted to that aesthetic, to share in the process. Cultural capital, in the form of mathematical aesthetics and knowledge, led to social capital.

As Bourdieu (1986) describes, there can be a relationship between social and cultural capital where one can contribute to the acquisition of the other. The accounts of these mathematicians amount to successive moments where particular kinds of social and cultural capital support the development of the mathematician's process. In discussing their experiences on either side of the advisor/advisee relationship, these faculty members highlighted ways in which an advisor potentially affords (or does not afford) social capital to their mentees. As advisors frequently serve as their students' early collaborators, forge introductions, facilitate opportunities, and create points of entry to a mathematical community, the faculty members not only contribute to students' social capital, but model its value and instruct the students in methods of creating it.

This social capital then served the mathematicians very well, affording them entry into conversations and social arenas where they could make further contacts, be exposed to more mathematics, or find questions to pursue: the social field of mathematics (per Bourdieu, 1986, 1993). The social capital of the advisor or other mentors can evidently be one of the more important assets of a training program in this mathematical creative knowledge generation process. In addition, mathematicians who are well-connected socially may be able to draw on their connections to provide their students relevant dissertation questions and then guide them through the process of solving them, and they may provide contacts who offer insights or share new ways of working. These contacts continue to be important throughout their career. When asked how she found her collaborators and mathematical community, Math Nine replied, "That's a good question, and I think that's one of the hard things for young mathematicians, and it can make or break someone."

Meanwhile, the mathematicians developed a sense of the aesthetic values underscoring mathematics, building this sort of cultural capital through exposure, which provided access to more social connections, which then, in turn, brought them into conversations that could help further refine their aesthetic sense. And through graduate school and perhaps postdoctoral experiences, they also gained access to the method of finding a question and the communities with whom to investigate and share those questions. In this way, these participants revealed how mathematics is socially-constructed by admittance to the canon-making arena through social connections, influenced by shared ideas of cool, beautiful, or interesting ideas, that are important to participation co-creating new math.

And as the students become practitioners, they may both collect more capital and become valuable contacts themselves, contributing to the self-perpetuating process Math Seven describes whereby her advisor has access to a great many ideas as a result of collecting many successful people around him, creating more opportunities to recognize open questions for himself and his students, and creating opportunities for those students to contribute to the field, creating more of a reason for others to be drawn to their successful community, which can collect more ideas, and the disciplinary conversation continues.

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Paper 3: Informally Learning the Creative Research Processes in the Social Spaces of the Studio, Lab, and Field

Abstract

The physical sciences, driven by curiosity about the natural world, have long been part of a liberal education, and the fine arts, while not originally part of the liberal arts, are disciplines in which curiosity guides an exploratory process. These disciplines are arenas of creative work, where practitioners engage in the creation of new ideas through processes of inquiry and investigation. Starting from the perspective that curiosity-driven pursuits are an important part of the human experience and engage the intellectual skills of attuned perception and inquiry, and the ability to flexibly consider complex ideas from multiple perspectives– key goals of a liberal education– this study is part of a larger investigation that asks how the creative knowledge-generation processes are imparted and developed in the curiosity-driven disciplines in higher education. The present study queries specifically how, among artists and scientists, the process is developed informally in the social environments of the lab, field, and studio. Drawing on 16 faculty interviews, this paper overviews emergent themes regarding the ways these social environments influence learning and practice, and then looks at themes across the spaces, including dynamics of informal learning, in-groups and out-groups, and issues of gender.

Paper 3: Informally Learning the Creative Research Processes in the Social Spaces of the Studio, Lab, and Field

Recognizing curiosity-driven pursuits as the historical foothold of liberal education, this research emerges from a perception that curiosity-driven liberal arts disciplines occupy a diminishing space in the academy. The physical sciences, driven by curiosity about the natural world, have long been part of a liberal education; astronomy was one of the original seven liberal "arts" intended to cultivate the practice of liberated thinking. The fine arts, while not originally part of the liberal arts, are pursuits in which curiosity guides an exploratory process. These disciplines are arenas of creative work, where practitioners engage in the creation of new ideas through processes of inquiry and investigation. Starting from the perspective that curiosity-driven pursuits are an important part of the human experience and engage the intellectual skills of attuned perception, inquiry, and the ability to consider complex ideas from multiple perspectives– key goals of a liberal education– this research asks how engagement in the process of creating new knowledge in the curiosity-driven disciplines of fine art and the physical sciences is imparted in higher education, with a particular focus on the informal learning that happens in the social and solitary spaces of practice.

The definition of the creative process used in this study draws on Dewey's (1934/1980) description of the act of making new meaning via active perception and a transformative "doing and undergoing" in investigating and experiencing that perception. This experience of creative production, in which reflection upon the phenomenon of interest brings about new knowledge, creates new meaning that transforms both the individual and the object of reflection; this process is iterative and ongoing. This resonates with accounts of the artistic process by contemporary artists, and since Dewey suggests that his description applies equally to science (also summarized in Hallman, 1964) this exploratory process is used as a starting point for this study regarding the processes artists and physical scientists.

Summary of Background Literature

The spaces of artistic and scientific practice in higher education are imbued with layers of the process: the equipment and furniture, places of solitary work, and areas of group meetings and presentations. They may be inside or outside, and may include multiple places that are local to each other, or more distant sites. More importantly, though, they are the places where the behaviors and dispositions of the creative processes of these disciplines unfold and can be seen, where people’s priorities and values are made evident, and where relationships are formed over long hours, excitement, and challenges. These spaces may offer students the opportunity to observe and immerse themselves in the creative knowledge-generation process of the discipline, building the relationships with those who will impart the process.

For instance, field-based learning has traditionally been a cornerstone of training in the geosciences, and Mogk and Goodwin (2012) elaborate on the ways time in the field serves to invite (or initiate) students into the "practitioners' wisdom" through embodied learning-by-doing, noting that this sort of responsive learning has a beneficial, affective component and is not replaceable with a more predictable lab-based curriculum. Complicating this picture, though, Núñez, Posselt, Hallmark, Rivera, and Southern (2019) report on how field-based learning in the geosciences can contribute to the social exclusion of minoritized students, who can feel like outsiders within a disciplinary subculture.

Shields (2018) looks at artists' studios as places of research, of not knowing, and of experiential and relational learning, directing her findings to thoughts about studio experiences for students. The art studio can offer students the opportunity to fully dive into the process of artistic inquiry, including artistic risk-taking and perseverance, and can also teach students how to exercise multiple points of view, as they interact in a studio community (i.e., Gude, 2004, 2007; Salazar, 2013). Meanwhile, Salazar (2013) finds that the personality of the professor contributes significantly to the learning environment, which raises questions about the role played by social fit.

When the process is imparted in the spaces where knowledge is generated by practitioners, a form of tacit knowing may be shared from practitioner to student; knowing may be made available through doing. But while learning in the spaces of practice provides a unique way to immerse in the epistemology of the disciplines, these spaces are not universally inviting. Perez (2016) discusses how broader cultural factors can dissuade underrepresented students from pursuing physics, as can the competitive culture of the discipline. Reporting on the experiences of two doctoral candidates, Gonsalves (2018) discusses how gendered cultural norms in the spaces of astrophysics can contribute to inclusion or exclusion and affect the formation of science identities. Intersectionality complicates this further, as discussed by Núñez, Rivera, and Hallmark (2020) with regards to geology, and Rosa and Mensah (2016) describe how the environments of physics can present particular challenges for African American women.

This raises additional questions and concerns regarding how the knowledge-generation processes are made available to students in these spaces.

Education in the creative knowledge-generation process in these disciplines incorporates the immediate social environment of a community of mentors and peers and the broader discipline's canon and methods. This paper focuses how this social context affects the ways the behaviors of creative knowledge generation are imparted or developed in the studio, lab, and field, inclusive of their dispositions, ways of knowing, and aesthetic sensibilities. It also queries how the invitation to engage these processes is extended in these environments, inclusive of the traditions and aesthetics of the disciplines. Drawing on the experiences of faculty members, this paper seeks to understand how the processes of making new knowledge in art or science are imparted or developed in the social environments of the studio, lab, and field.

Methodology

This present study is a part of a larger investigation with faculty participants in curiosity-driven disciplines in the arts, physical sciences, and pure mathematics to query how they learned and teach the creative processes of their disciplines. The current study is only concerned with the physical scientists who trained in, and now run, research groups and with artists who trained in, and now oversee, students in studio environments.

To investigate how artists and scientists develop and impart their creative research processes, I conducted semi-structured interviews with six artists and ten physical scientists who are engaged in both teaching and research. The semi-structured interview format invited participants to share their experiences around a few broad questions and also created space for unexpected ideas to arise and to be clarified with follow-up questions that drew on details provided from within the participant's narrative (Bogdan & Biklen, 2006). This allowed the interviews to focus on, and further query, the teaching, education, and research experiences the participants described, which is in line with the interpretivist paradigmatic stance of this research, seeking to understand the meanings of the participants' accounts through rich, personalized narratives (Lather, 2006; Schwandt, 2000; Pascale, 2011). Interview data were analyzed to both look for findings addressing the primary, exploratory research questions and to attend to ideas emerging in the data.

Participants

In order to address the ways that creative processes are imparted and developed in higher education, the science participants for this study were faculty members in the disciplines of earth and planetary science, the geosciences, astronomy, and physics at research universities, and who are actively engaged in research and teaching in research group or team settings, as confirmed by faculty websites and course listings. The artist participants teach at the higher education level and have active artistic research practices in a range of contemporary and traditional formats, as evidenced by exhibitions or active projects; they include painters, mixed media artists, and sound artists. Because this study focused on curiosity-driven disciplines, it did not include faculty from applied fields. Science participants were from four U.S. colleges and universities; artist participants were from four different colleges or art schools. The participants were also chosen to include faculty members who were at different moments in their career and who pursued their education and entered teaching at different times. They ranged from those very well known in the discipline to newer faculty who were just getting established. Participants had received their PhDs or MFAs in the 1980s through the 2010s.

In light of the gender disproportionality in the physical sciences, I made an effort to include 50% women or non-binary individuals. In order to achieve this goal, I contacted women at more than double the rate of men, and at a greater number of universities. Women were few among the faculties perused, particularly in certain sub-disciplines (i.e., theoretical physics), and gender diversity skewed younger, particularly in the field sciences. While an effort was made to recruit participants from a diversity of backgrounds, there were so few faculty from underrepresented minorities on the websites of the departments considered that none of the faculty interviewed contribute to the racial diversity of the participants; this lack of diversity will be addressed again in the discussion.

Recruitment

Selected faculty members were emailed at their university email addresses and were provided a brief description of the project, asking whether they would be interested in participating. No financial incentives were provided. To pursue recruitment goals, faculty were contacted in sets, awaiting responses before contacting additional faculty, responsive to the evolving list of participants and the topics and backgrounds they brought to the research.

Overall, I emailed 41 scientists, 15 men and 26 women; 26 were in departments of physics and/or astronomy, and 15 were in departments of Earth and planetary science or geoscience. The response rates by discipline and gender ranged from 7% to 75%. The lower response rate for women was possibly due to the frequent asks women in the discipline receive. All of the interviewed faculty in departments of geosciences or planetary science were trained in and oversee research groups. In physics or astronomy, a subset of the faculty, one woman and two men, oversee research groups and were trained in similar settings. This amounted to ten interviews with faculty whose training and present practices involve science teams (see Table 1).

Table 1.

Scientists contacted and interviewed by gender and discipline

(Earth, planetary, and geo-sciences or physics and astronomy)

	Total	EPGS contact	EPGS interview	PA contact	PA interview	Team PA interview
Women	26	11	4	15	1	1
Men	15	4	3	11	5	2
Total	41	15	7	26	6	3

Response rates for fine artists were much higher, and the goal number of participants was lower due to my greater familiarity with artistic disciplines and the availability of a relatively rich literature. To calibrate previous knowledge to the current study and questions, the goal was six artists. Seven agreed to an interview, three additional artists did not respond, and six interviews were successfully scheduled.

Interviews

The approved interview questions addressed three primary topics: How the faculty members learned how to engage in their current creative/research process; their description of their process; and their goals in imparting the process to their students. Additional approved questions addressed idea generation, investigation, or experimental processes; mentorship or advisorship; autodidaction; socio-emotional dispositions (often rephrased in the interviews as “attitudes to the work”); aesthetics; and social environments. If these topics did not come up in the course of the conversation, I asked follow-up or protocol questions on these topics.

After several interviews, it became clear that the protocol was too long. Extending the notion of emergent design (Jacob & Furgerson, 2012), which I was employing to choose relevant protocol or follow-up questions to ask within the interviews, I chose to revise the protocol in a more deliberate way by honing in on the most pertinent questions. I reviewed the early data, paying attention to which questions were arising naturally and which were yielding rich descriptions, and I reprioritized the protocol, leaving many questions off the list during subsequent interviews.

After an early review of half of the data from the full study (including the mathematicians in the full study), it became clear that informal learning and social environments were a significant part of the participants’ experience, and I chose to add a broad question regarding interviewee’s experiences with bias and/or privilege in their learning, teaching, or research environments. The second half of the interviews with artists and scientists included this question, and I contacted most of the previous participants to invite them to a follow-up conversation on this topic (some had already addressed these issues in the course of the first interview and were not contacted for a follow-up.) Five participants agreed to a second conversation.

The interviews themselves were semi-structured. Each interview addressed the main questions of the protocol, while the conversations were allowed to flow organically within each question. In some cases, I asked a number of secondary and tertiary questions from the protocol, and in other cases, I asked only the main questions and follow-up questions based on the participant's responses. Participants naturally had different conversational styles, and some responded more to a series of shorter, discrete questions, while others spoke more narratively in response to broad questions. For the purposes of this exploratory research, allowing for this variation in conversational style allowed the participants to provide context where they wished, while leaving the option follow-up questions to deliberately pursue subsequent questions for greater detail at other times. One limitation of this approach was that, due to the time-limited nature of these interviews, some topics were not covered in all conversations. An emphasis was placed, instead, on thick description and on follow-up questions to pursue tangible and immersive detail, such that I could picture the situation the participants described, which is referred to as crystallization (Tracy, 2010) and is seen as an alternative to triangulation in pursing clarity, and arising with a greater acknowledgement of subjectivity.

Interviews were conducted in person, over video call, and over the phone. They lasted between 45 and 90 minutes. Prior to the interview, I reviewed the consent form with each participant. For video and phone calls, I asked for verbal consent including three yes/no questions that addressed recording and transcription. For in-person interviews, I provided a paper consent form, which we both signed. When consent was given, I audio recorded the interviews; I made an exception for some phone calls, preferring to take notes rather than record as recording the call decreased the audio quality during the conversation.

Analytical strategy

The interviews required in-process moments of reflection and analysis. Because of the abstract nature of the constructs queried, it was often necessary to present my interpretation of what an interviewee was saying, as a summarization of points they had made, either to contextualize another question, or to ask them to correct it. These in-process analytical moments both introduced and removed bias from the interviews by making evident that my interpretation and priorities were guiding the conversation, particularly in follow-up questions and by deliberately inviting participant checks on some of those interpretations. These moments increased the conversational tone of the interviews.

I transcribed some of the interview audio files directly, and I cut other audio files for anonymization and relevance and sent them for transcription to a company that had provided a non-disclosure agreement, later validating the transcript. When I took notes instead of recording, I immediately reviewed them and added details from the conversation. These processes served as a way to become familiar with the data and reflect upon it. The resulting transcripts were then iteratively reviewed, with sections highlighted, categorized, and annotated in comments; these annotated transcripts served as working documents for emerging ideas.

I organized data by main topic in a spreadsheet early on in the process. This spreadsheet allowed for a quick glance at which questions were yielding rich data. This worked particularly well because the intended topics of the interview questions did not arise in tight, clean units; they were often alluded to across multiple different responses. The spreadsheet allowed me to visualize the topics covered across all interviews. I often summarized the data in prose, sometimes including quotes. The resulting spreadsheet summarized data by topic code (per Saldaña, 2013) and participant.

Once the interviews and dataset were nearly complete, I began to code the data using both Atlas TI and Quirkos. I realized part of the way into this endeavor, though, that by iteratively reading the files and organizing the data into the spreadsheet, I had topic-coded all the data and already arrived at most of the cross-topic themes of this paper. Realizing I was duplicating my coding efforts to justify results I had already obtained, I discontinued this practice. Instead, I continued to employ the spreadsheet of data, and began writing themed memos to analyze and explore emergent ideas more holistically.

Iterative writing served as a final and important part of analysis. I reviewed, compared, and annotated memos; created lists of emerging ideas; and utilized spreadsheets to organize complex, interrelated themes. I iteratively moved between writing that reorganized data and descriptive prose, reviewing the data continuously in the process.

Findings

The faculty members offered accounts of the ways they learned to engage in their current creative knowledge-making process, what that process looks like, and how they impart it to their students. Due to the semi-structured and exploratory nature of the interviews, the conversations included the unique experiences of the individual faculty members across moments in their careers. They discussed learning from peers, working independently, exploring out in the field, the presence or absence of their advisors, and their efforts to foster supportive environments for their students. Within these varied accounts, several related themes emerged regarding the informal and social learning of the processes in the spaces of the practice– the lab, field, and studio. These themes are discussed below, presenting parts of the narratives from which they emerged.

Primer: The Creative Process is Practiced Differently Across the Disciplines.

These interviews were based on a model of the creative process that includes idea generation or question formation, followed by iterations of investigation, reflection, and revision, which might alter the initial idea or question. This model was derived from descriptions of the artistic process and of creative processes more generally (see O'Grady, 2021). In the course of these interviews, I asked about the particular elements of the participants' processes to check that the model fit their work. For the artists, the model fit well. For the scientists, the model generally fit, but there were notable exceptions. Their questions were sometimes developed in looking at data or in responding to papers; they were sometimes developed in response to open exploration of the Earth or sky; and they were sometimes extensions of conversations with labmates or others in the field– all of this is consistent with the model of the creative process that I was using. But sometimes their questions were set externally, by funders or an institution. For instance, one physicist works on a longstanding project that existed before his arrival at his university, and the primary questions and general approach of this project have been in place for a long time. When asked when he started to do his own research, he explains, "In some ways I haven't ever done my own research, in the sense that I've always been a part of these large collaborations." And another scientist loves writing proposals in response to calls from funders, so his questions often follow from the specific research agenda of a larger institution and government interests. This changes more than the authorship of the question. It means that, during the creative process, those practitioners who chose what to ask may revisit and revise the question itself, but those scientists who work on externally set questions engage in revisions primarily to problem-solving approaches or possible reframing of the given question.

Additionally, the iterations of investigative behaviors– experimentation, reflection, and revision– addressed a range of textures of inquiry, from problems of instrumentation to building concepts. One scientist, in response to a question about whether the above model of the process fit to her work, said, “In principle it would be nice if that were the case,” but she explained that because her work relies on intermittent satellite data coming back to Earth every few decades, the iterations of reflection and revision respond to little new information, “so it’s iterative in some sense, but it’s like you’re playing a really, really long game.” This is very different from the iterations of reflection and revision, or the even more immediate reflection-in-action, seen with artists or mathematicians, and it clearly changes the behaviors of the process, which would then change the ways the processes are imparted from “master” to “apprentice,” in higher education.

Finally, Physics Four asked to save time at the end of the interview to assert that she did not think of herself as creative, instead referring to a logical list of strategies that she has developed in her process. This was somewhat echoed by Earth One, who made the point that while she was concerned early on about how to learn this kind of creative process, she eventually learned that “it is a system. Even though it’s creative, there is also *a system*…” When I asked Physics Four whether we could make the distinction between “a creative person” and a person who has a creative process, she still did not think her process was a creative one. She instead referred to the common pairing of the ideas of genius and creativity and noted that fields that are commonly regarded as creative or genius have much lower rates of participation by women and underrepresented minorities (for example, see Leslie et al., 2015).

Social and Solitary Learning

The ten scientists represented in this paper all studied in, and now mentor, small research groups. Earth One explains her choice for this kind of team. “I knew that I could actually make a contribution, that I could have my own thoughts and work on those and figure it out, as opposed to being one person in a really large endeavor.” Comprising somewhere between a few and a dozen people, these groups are unified by methods, areas of interest, shared equipment, or contributions to a larger scientific project. Within these sorts of environments when they were students, the scientists had developed their processes in a combination of social and solitary ways, developing technical skills, methodological understanding, and content knowledge through peer learning, self-teaching, modeling, and informal mentorship. They also often learned by doing, sometimes by being “thrown into the deep end.” Their social learning environments, whether in the lab, office, or field, provided different types of structure and involvement of mentors that shaped the way they learned.

The six artists all studied in and now oversee students working in studio environments. And while the scientists primarily discussed graduate school experiences in the development of their processes, when the artists talked about how they started to engage in their own creative work, they all mention an earlier time, either in mid-undergraduate studies or before. They weren’t thrown into the deep end, because to a large extent, they had chosen to navigate artistic waters, with different levels of intensity, for a long time. Still, the social and solitary elements of studio life, where they spent long hours with other art students, played a large role in their learning and the development of their artistic practices.

No Formal Way

When asked how they developed their processes, several of the scientists immediately detailed the lack of formal instruction in their learning experiences. Physics Four succinctly responded, “No formal education,” elaborating later that, “We don't teach people how to do research per se, so for me at least [it] was a lot of watching and emulating…” Physics Two states that you “learn as you go … there is no formal classroom training.” Earth Four asks:

> How do you develop research questions, and how do you learn how to be a creative scientist, and how do you learn how to take risks? … some of that just comes with experience, and I can't tell you that there was any single moment or any single person that taught me how to do that.

Physics Seven summarizes, “It just happens,” adding, “I think it’s learning by observation, experience, and participation.“ He also discusses how an emphasis on coursework is not helpful to learning the process, saying that courses teach the science content, but “Our job, when we educate PhDs in physics, is not to teach them science. It’s to teach them how to be scientists.” This prioritization places mentorship and the social environment in the center of a scientist’s training.

The artists’ responses stretched back to their youth and overviewed what was often a personal evolution of artistic interests. Self-teaching and self-guided explorations featured naturally throughout their descriptions. Unlike the scientists’ responses, there was no implicit teacher of the process present or absent, because it seemed natural that the process was an independent one. Learning by doing was generally a given. As they entered the studio environments of higher education, they experienced new forms of engagement, both socially and alone. The sections that follow outline some of the specific features of learning in the lab, field, and studio that emerged in the interviews.

The Lab

Watching Others in the Lab

Although only a subset of the scientists worked in the field, all spent some of their studies in a lab, where, in the absence of formal teaching, they describe thoughtfully observing the actions of faculty and more advanced researchers. Earth One calls herself a "mimicker" for the way she incorporates the processes of others into her own, evolving her practice over time. And Physics Four, who learned her process emulating others, explains that this approach was facilitated because she started out as a student assisting with a large, multi-institution collaboration, so she had exposure to a lot of scientists and could watch how they worked and approached problems. Later, when addressing questions of her own, she would think of them and wonder, "If I were so-and-so, how would they approach it?" and, as a result, she developed a menu of actions she could take in her own process.

One scientist describes how things "filtered in" while spending time in both her adolescence and in her higher education training around scientists, saying of the behaviors of the successful faculty she watched, "I'm sure many of the things [they did] were on purpose, but it was never clear to me which ones were on purpose and which ones were quirks of personality," so she "spent a … lot of time trying to figure out which thing they're successful because of, and which things they are successful in spite of." Now she explains the rationale behind her choices and suggestions to her students, telling them, "This thing that feels weird to you is on purpose, and I'm going to tell you why, and I may be right or wrong in doing it this way, but I at least want you to know that I'm doing it on purpose."

Modeling the Process

Like the scientist quoted above, after watching their faculty and figuring out so much of the process on their own, other faculty members also choose to mentor their students by transparently modeling their processes with their research group. Another scientist also deliberately makes her thought process clear for her team, essentially providing the teaching that she learned by emulating those around her, but in a direct way, "because I know how frustrating it was to try to do it on your own." She explains that she doesn't have all the answers, but she talks them through ideas to try when they get stuck, much like the menu of problem-solving options she developed for herself, elaborating that, "because I was never taught how to do research … [I] try to explain why I'm giving this suggestion or why I'm thinking about it this way."

One scientist summarizes her process as a "follow-the-data type mentality" and describes a lot of brainstorming to come up with interesting ideas about planets. When asked which parts of her process she models for her research group, she replies, "I think it's all shareable … in group meetings, we do it in real time … We sit there and think of some weird idea, and try and work it out, and decide whether or not it's feasible…" She also models not having all the answers and a disposition of being ready to give up an idea or a cool, new theory if the data don't support it. She says of learning to let go of an idea:

> I think that it's important to do that step as early as possible. Because you never know what you're going to get, so it's worth just throwing it out there and seeing if it works. I think that also demonstrates that it's okay to have an idea that doesn't pan out as well, right? It doesn't mean it was a bad idea."

She explains that she thinks sometimes too much emphasis is placed on getting the right idea, when a very cool idea might later be disproven, "but it was supported by the data at the time."

Earth Two describes the way he models his process during weekly meetings with his small research group, approximately half a dozen people unified by a common geographical and conceptual area of interest. As team members share their work and collectively discuss ideas and methods, he makes his practice transparent by demonstrating idea generation and explaining his choices in the research process. He guides students early in their career, explaining, "Notice the fork in the road; here's the reasoning…" and "then subsequently when they get more advanced, I would say, 'Here's a fork in the road, and you're going to explain why we think we should go this way.' I kind of hand it over to them."

Peer-to-Peer Learning and Hanging Out

One of the informal ways in which student members of a team learn is, unsurprisingly, from each other. Drawing from the same mentorship and research environment, they share tools, methods, content, and questions. Earth One describes one graduate school teammate as instrumental in her learning, particularly in working on the computer: "I wouldn't have gone anywhere without him… There was a lot of peer-to-peer mentorship at that level." She also enthusiastically describes how she has "the best group right now … they're very collaborative. They're writing papers together, they're working on stuff together … This is just them hanging out together and working together.” Another scientist draws a clear picture of the enthusiasm of her group getting excited about “weird things they learn about planets” and the surprise and brainstorming that result as teammates try to figure out new data together. The research groups attract members with similar interests, and so their exchanges often center on shared curiosity.

Physics Four recalls how she and the rest of her advisor’s research group had to figure out a good deal together without faculty involvement, solving problems and coming up with ideas, describing how “the best ideas” come from interactions when people bring different backgrounds to a question. They worked together to such an extent that, by the time she graduated, she felt, “given their slides, I could have given their [research] presentations for them.” She now makes this expectation clear for her advisees: she wants them to work together enough that they can give each others’ presentations, and she will redirect students’ questions to other members of the group, not only because she has limited time, but also for the benefit of learning from each other in this way. One theoretical physicist notes how it is standard for students, postdocs, and faculty to learn a lot from each other, describing how they are “always working with different people on different projects,” a few at a time, and how the arc of projects is short enough that students can see several projects to completion during their PhD and work with many combinations of peers and mentors. He says of his sub-discipline, “It’s a much more social science than you’d think.“

One geologist, now at research center where he and his colleagues share advanced geological instrumentation, describes the importance of knowledge-sharing in his own graduate experience and in that of his current students. He observes that peer exchange and cooperation may have increased over the years, suggesting the increase may be because the degree of cooperation and collegiality among the faculty advisors sharing the facilities is greater today, and faculty will encourage students to interact with their colleagues, contributing to a greater sense of community overall. Another scientist at a different university mentions something similar, noting that students often report to multiple faculty in the same building, which contributes to cohesion in the department as a whole, from which the students then benefit.

Learning Alone Together

Within the lab and research group environments, students are expected to learn or figure out a lot on their own, often in the company of others. One scientist says that it's "typically the scenario in our field … you're expected to do a lot of your own learning, then just be guided throughout the process." She taught herself a lot of content, starting early with space magazines in her teen years, and she now models self-teaching with her group by sharing her self-teaching goals. One Earth scientist describes how basically none of the content or methods he needs for his process were part of any formal instruction, so he taught himself a lot; this was echoed by participants across the sciences. And Physics Two says, "it becomes expected of you, and then is the only way." This self-teaching is not uncommon in higher education, whether motivated by curiosity or academic pressure, but what is notable here is that some of the scientists also discuss learning the arc of research and the behaviors of questioning and problem-solving–the creative knowledge generation process–with little guidance.

For Physics Four, collaboration and self-teaching are “inextricably linked” because she had so little instruction in research, saying of her graduate school research group, “Everyone's trying to figure out something together, and everyone knows different parts of it,” describing the value of a diversity of perspectives. She also draws a distinction between the things you just look up, like writing a type of code block, which she does not count as teaching herself, versus learning *when* to look something up and *when* to keep at it, making the point that this understanding is more important, and she self-taught those methods, too.

One geologist arrived at her PhD program with an undergraduate degree in a related science and quickly realized that a good deal of geological knowledge was assumed already, thinking to herself, “I gotta learn all this stuff,” recounting that it was a lot of work. But beyond content, she also taught herself a research methodology, saying, “The research process is very much like … your own detective sort of thing.” She tells the story of how her advisor had initially given her a research topic that suited her interests, but when the first try did not yield results, he took her off the project and brought her to a cabinet of rocks, telling her that those were to be her dissertation. Having little education in rock types or related research methods, she was surprised, and she went to the university bookstore and bought a book. “I wish I still had it, but it's how … to do research, what is it about, and I read the book.” From that book, she independently taught herself a good deal of the research process.

Peers also rely on each other to figure things out on their own when they are contributing to one research agenda. As one physicist describes, “It’s a culture where you’re not alone, but you’re … figuring things out on your own, and … there’s a reliance on you to do that." He also says of the group he runs now, "People are kind of left on their own to figure things out … with some amount of guidance from the postdocs and the more senior students … you depend on other people for what they're doing, so there's a strong team cohesion, but … nobody gets pulled along, everyone has to run, and then you run together." He elaborates on the intensity of this environment, "We have had people who start and just can't manage it," explaining that they might leave the group. As a student, this "fiercely independent and competitive" culture helped to motivate his independent learning, but this differs from descriptions of the work cultures with which some other participants were comfortable.

Middle-Mentorship

As the physicist quoted above indicates, within this informal learning lab environment, advanced graduate students or research associates often provide mentorship to students earlier in their careers. When he was an undergraduate in a lab, graduate students mentored him, thereby modeling their approach to work, and this continued into grad school. He describes the way the lab environment felt "familial," noting that, "It's an apprenticeship, right?" This social arrangement influences the way he runs his lab now. "I do depend a lot on the graduate students and the postdocs to do a lot of mentorship."

One geologist describes how the extensive, supportive interactions among graduate students and postdocs in his current lab are "an invaluable part of the experience for both of them." He also describes how one of his PhD candidates has been instrumental in recruiting undergraduate students, whom she also effectively mentored, adding that his lab has a "postdoc-graduate-undergraduate group … that works really well together,“ and he is “super pleased by it.” In contrast, as a graduate student, another geologist spent a few months working closely in the lab with a postdoc who was helping her with her research. As the postdoc became more of a mentor, her advisor made a point of speaking with her to say, as she puts it, “Let me set you straight, I’m your advisor,” clarifying that the hierarchy stopped with him, not the postdoc.

This middle-mentorship format requires flexibility in response to the personalities in the lab. One physicist explains that he has recently stepped in more often to arrange direct meetings with students when the informal mentorship between postdocs, grads, and undergrads hasn’t happened, describing “a little bit of a social breakdown” in which “the group is not as socially capable at this point.” If middle-mentorship requires a socially cohesive group to work, it is worth asking what it requires to be equitable. Given how much of lab knowledge is to be informally imparted or learned experientially, access to informal mentorship and shared experiences directly impacts students’ learning and engagement in the process. Experienced members of the group, by interacting informally with those students with whom they feel most comfortable, may affect the way that certain team members differently experience, practice, and learn to engage in the process. Where the social and cultural capital of the “master” is conveyed to the “apprentice” via a middle-mentor, there is the possibility of additional bias and privilege entering into the situation. Cultural fit then potentially becomes a filter for educating a next generation of researchers.

The Field

Out in the Field

Perhaps the learning arena presenting the greatest range of social and solitary experiences is that of the field. Driven by a curiosity about the Earth, geologists go out in the wilderness, camping and hiking to study particular sites. In these environments, the presence or absence of mentorship and preparation seems to affect the constancy of quality and equity in learning, and has implications for emotional and physical safety. This is not to say that direct mentorship leads to higher quality, but rather that the range of possibilities increases as structure lessens. The geologists in this study discuss their experiences in the field, which range from the enviably enjoyable to the dangerous and miserable, seemingly hinging to some extent on the level of direction and preparation, and the attitudes towards women in the field.

One geologist remembers her "lovely'" undergraduate experiences: "It was [studying] minerals and going camping … with a bunch of friends and enjoying life..." Later, she elaborates, "We were always together as a group, and we walked with the professor, and he would explain what he sees, and you would take your notes … and it was so nice and simple." Similarly, another geologist had great times doing labs and going on field trips with some of her closest friends in a small group of undergraduate geology majors. As part of her senior thesis, she had the opportunity to collect her own samples in the field. When a professor explained that she could make a career of it, "that blew [her] away."

Earth Five was attracted to geology in large part due to the field component, transferring to the discipline from another outdoors-based major; he also describes enjoying his undergraduate geology fieldtrips. He now provides popular field trips to undergraduate classes, sometimes taking his students to places where he knows there are "cool features," but he adds that it's also useful to go to new places with students, "to figure things out on the fly because you're seeing it for the first time yourself, and they get to experience that process." This resonates with research suggesting that one benefit of fieldwork is the imparting of "practitioner's wisdom," as students observe the real-time active thinking, affect, and immersive learning of experienced mentors (Mogk & Goodwin, 2012).

The Wilds of Grad School

The geologists' fieldwork experiences started to diverge in graduate school, as they found themselves at times in challenging environments. In contrast to the cooperative and collegial nature of her undergraduate field work, one geologist describes the behaviors of her grad school peers in the field as "just so competitive… they didn't work together…. it was like this," she gestures with her hands a quick dispersing of people away from each other, as though competing to see who would "get the most lines on their map." Another geologist began her graduate program by working "in the wilderness for a month" with "no preparation… no packing list, no mental preparation." She describes this as "an extreme situation, so I was very much thrown in the deep end there," noting that, "there's a lot of room for improvement."

During their graduate studies, each of the geologists spent a good deal of time out in the field, unsupervised. Earth Five spent many months at a remote field site alone, in a "remote, rugged, isolated area … with a lot of frontier-like characteristics." He had occasional visitors during this self-directed experience, including one visit from his advisor, but he estimates that for between a third and half of the time, he was solitary. He describes this experience as "wonderful," as he would spend his days in the landscape, making a map, collecting samples, and going back to his camp. It was one of the defining times in his life: “This was it for me.” But he faced some daunting encounters outdoors, even though he had spent enough time in the woods to be very comfortable. He makes it clear that “it isn’t about tolerance” for being outside in the field. “It’s about *I love this. I wouldn’t do anything else*.”

One geologist went to an international field site with another of her advisor’s female advisees, and she describes a good relationship in which they collaborated well. But she also says, “there were some things I wasn't prepared for… safety concerns …walking around cities by yourself, being out in the field … sometimes I would kind of go off on my own,” saying that some bad experiences now inform her desire to prepare students better for fieldwork. And another female geologist was encouraged to deepen her fieldwork experience, so she joined with a male student for three months at an international field site, and after a short time, he wanted her to leave, resulting in many uncomfortable experiences while they were alone in a rugged area abroad. But regardless of the miserable times, as she describes one particular day when he insisted she carry all the samples in a heavy backpack full of rocks, she wouldn’t leave. She instead felt blessed to be able to touch these amazing places where geologically significant events in Earth’s history began.

Although all of the interviewed geologists love the outdoor components of their processes and the explorations and discoveries that happen there, the unstructured nature of the field put their experiences in the arena of personality fit, the whims of nature, and the unknowns of a foreign place, resulting in a range of wonderful and challenging experiences: There are many ways this learning environment can unfold. Two female scientists also went to the field with female mentors from other institutions at the recommendation of their male advisors. And in contrast to graduate experiences with competitive peers and challenges in international fieldwork, one of these scientists describes a wonderful experience, “how it should be,” with her female mentor at the lead and two other geologists on the team. The four of them would be together, “looking for something … you’re cooperating and … you're discussing what you're seeing, and you're trying to learn, and you're enjoying each other's company...” Adding to the point that these field experiences can go a number of ways, she concludes, “... so then you realize that not every field experience is the same … and so [you] don't need to be discouraged” by the unpleasant ones.

And it is essential that students are not discouraged from field experiences for the unique learning that happens there. As the Earth’s storytellers and story-finders, they discuss the importance of “putting in the time” to learn how to *see* the story, noting that deeply understanding a local story allows them to connect it to a broader narrative of the Earth’s history. One geologist describes how members of her discipline develop a way of seeing and reading the land that is unique and mentions a mentor, saying, “She had *the eye*,” illustrating this with the point that she could walk over and find something significant right at your feet. The development of this lens of the field geologist, the “eye” to be able to see a particular type of ancient landform or an arrowhead at your foot, takes time out in the field to develop, which places additional importance on the quality of a student’s or researcher’s field work experiences and the support and motivation necessary to stay there.

Drinking in the Field

In field geology, where discouragement from fieldwork means discouragement from learning the process, the reputation for a biased environment is strong. One scientist from a related discipline discusses field geologists in her broader academic community, describing a “branch that is very macho … ‘I'm going to go out into the desert … and figure out how the universe formed and drink a bunch of beer and tell a bunch of sexist jokes around the campfire.’ ” And even now as a faculty member, one female geologist will not go into the field with certain university classes or colleagues, mentioning by way of example a trip with a class when the first stop was to the liquor store, and as cases of beer were added to the van with all of their luggage, she recalls feeling, “I’m not here for this. I’m not up for this kind of drinking situation.” She has worked in countries where drinking was prohibited, which “was wonderful,” and then in others where drinking was a cultural expectation, which “can cause a lot of problems.” She confirms the stereotypical image offered above, saying how it’s a “big time” tradition in the mostly-male geology community to go out and drink in the field. This could understandably dissuade some people from fieldwork.

The Studio

The studio environment is one where artists often work independently around central spaces, galleries, sinks, and equipment. Like a lab, it is where artistic investigation often occurs, and like a field camp, it is where artists convene to share their explorations. But the studio environment uniquely includes a potentially greater diversity of approaches and goals among community members. Whereas the scientists’ teams are unified in their work, often drawing from pre-set questions, content areas, or broad goals of their sub-discipline, and often center on shared equipment, the artists independently author their own questions starting at an early point in their career, leading to a greater range of pursuits in the community.

The Studio as Place for Being Curious

For these artists, curiosity drives both the question and the nature of the quest itself. The process is one of learning, wonder, and discovery: that's the goal. When asked what dispositions student need to bring to their learning to engage and develop the artistic process, Artist Five rapidly and clearly says "curiosity," later adding resourcefulness, which she feels she can teach, and elaborating that she doesn't feel like there is much she can do when curiosity isn't present. She describes how she started to make her own work during an independent study in undergrad that gave her "total freedom to explore whatever [she] wanted and investigate whatever [she] wanted," in which she drew from influences and interests in the arts and culture, reaching across different topics she was studying at the time to develop her artistic research. And Artist Two says that, "… one of the core motivations … for me when I'm doing this kind of work … it's an opportunity to express my curiosity for the world," asking, "what's the world presenting me?" He similarly invites his students to "start from a perspective of curiosity and openness" and "give yourself the time and space … and empower yourself" to engage in the process.

Artist Six explains that wonder is a significant motivator in his process, which essentially involves fieldwork, describing invigorating artistic research explorations out in the world, which create "a good excuse to make a painting" back in his studio. Discovery is a big part of this work, as he observes and collects what he sees, noting that "just moving slowly through the landscape you discover something." Similarly, Artist Five describes following an idea down an artistic path, even if it is not related to the original premise of the work, "but you just go anyway because … that's excitement, right? The discovery."

Artist Two discusses the way the studio environment can support this curiosity and discovery, saying that if the atmosphere is positive, it contributes to the learning and "is an important ingredient in learning healthy approaches to being a creative person." He elaborates that if the atmosphere is negative, for instance creating competitiveness or undue pressure, it can have a detrimental effect and teach bad habits to students, demonstrating the importance of an artistic studio community that encourages curiosity and presence rather than outward achievement. Artist Three finds additional motivation to engage in curiosity and discovery in the studio community, where "there's lots of people who think … that getting up and working on these projects all day is a valuable way to spend your time."

Studio Support and Exchange

The artists interviewed here describe how, when they were students, the process of entertaining and exploring their curiosities was supported by a community of other art students who were working differently from one another, but who were similarly exploring their own content through the artistic process. It was this shared spirit of exploration that shone through in these conversations, drawing a picture of the studio as a community supportive of artistic inquiry and experimentation broadly.

In her undergraduate experience, Artist Four shared a group studio with "a crew" of dedicated art students, and she could count on them to be working there. She describes a fun and supportive environment in which to develop her artistic practice, recalling getting a giant cookie and a coffee and heading over with her studio crew to work. Artist

Five is still in touch with a close friend from her graduate school studio group and continues to exchange feedback and ideas with him. As a student, she did not depend on the male faculty, whom she describes as “macho” and “sexist,” but she describes her peer community as:

> … a handful of people that I became very close to … basically we had we had critiques and exchanges all the time, and … they ended up being my support group for the rest of my life. I mean it was incredible” … “everyone worked, you know, all night, all day … it was just fabulous … the work ethic, too, that was really instilled by this group of people. We were all just crazy, just making art.

Both Artist Four and Artist Five describe similar communal studio spaces for the art majors where they currently teach, intended to provide supportive environments for their present students. When Artist Five advises undergraduate students about graduate school, she tells them, “… it's always nice to go somewhere and … know who you're gonna work with … but it's that community that you have that’s so important.”

Artist Six encourages his students to work together, telling them they will learn more from each other than they do from him. He describes this peer learning as an “unseen curriculum” in the studio building where he teaches; student artwork from intensive classes hangs on the walls and peers visit each others’ classrooms, which he says is very impactful for a lot of students. He recalls having a similar peer community when he was a first-year art student, staying up all night with classmates to complete projects and really enjoying it, saying this intensive shared experience is “probably what made [him] go down the path of art to begin with.”

Artist Six also describes the impact of peer support in his graduate school experience, even though he worked with the door to his private studio closed. "It was more the nature of being in a group where you share what you do … You're in this forced community… You have to go to their critiques, they have to go to yours…" He expresses particular appreciation at how peers "entertain your whims," which led him to his current process. "I don't think I would be doing what I'm doing if I had just had a studio, by myself, and I didn't have that same kind of community to bounce ideas off of or present work to."

Artistic Difference in the Studio

One artist chose her graduate program because it was immediately apparent to her that the social environment was stronger and more supportive at one program than at another she visited, and she had appreciated so much the community of her undergraduate studio environment. That group tended to have similar ideas about what art was, which she suggests perhaps was because they all had the same teachers, but during graduate school, she and her peers looked at all different artists, and it was "really eye-opening and fun to discover" her peers' different definitions and views about art, although one of her close friends and studio partners had little respect for the sort of realism that she pursued in her painting practice. Even with this disagreement, they got along well, and there was an acknowledgement that a diversity of approaches and theories of art was expected and supported in the studio.

The appreciation for different artistic styles and approaches can also apply to one's own practice and exploration. Artist One discusses how students need some confidence, bravery, and boldness to try new things artistically, particularly when they have already created a particular artistic identity connected to a certain genre. Another artist mentions how he moved beyond the limitations of one artistic form during his undergraduate studies, when he started working across as "many different art forms as [he] could" in "very fluid collaborations" with composers, choreographers, dancers, and video artists.

Mentoring Openness in the Studio

When mentoring and guiding students towards processes of their own, these faculty emphasized openness with regards to new ideas, experimentation, and their observations of the world. Artist Two is hesitant to describe his conversations with students as "feedback," noting the subjectivities of any response and not wanting to critique or influence a student's exploration. He instead prefers to provide an honest account of what he notices, allowing the student to choose whether to act on any of it. Artist Three says that when he works with students, he tries to "constantly [keep] them in a state where they can keep working and aren't encumbered by technological problems or a fear that what they're doing is wrong." He elaborates that it is "so essential to make them feel like they're in a safe place and … like they can do what they want to do and have the ability to do it … that's … amazingly important"

As their students learn how to make artistic choices for themselves and move towards authoring their own content, a number of the faculty also emphasize self-trust. Like Artist One, Artist Four is wary of being prescriptive to her students, saying that a lot of students "want to know the answer," but she teaches them that artists need to make their own choices. To lead them towards that, her approach is to:

> … encourage them to trust what they naturally start to gravitate towards, and trust that it will evolve and that they will have a deeper understanding of it through the process of making and reflecting on it … I think naturally everyone is sort of interested in … specific things that seem very obvious … because it's always been something they've been drawn towards, and it doesn't mean that our interests can't change and that we can't have new ideas or sort of go down different paths, but whatever they're naturally leaning towards … push into that and see where it goes ....

Artist Five teaches intensive skill building that moves into independent inquiry, with a goal that by their senior year or the end of an independent study, students can make their own assignments using the resources and assignments she has provided as a platform. To help them find a process of their own, she will direct students toward what she perceives as their strongest work. Meanwhile, Artist Six uses his own experiences to demonstrate to his students that the artistic process asks them to take risks and work outside of their comfort zone, noting that if he does his job right, the process becomes intrinsically motivating for his students.

Inconsistent Benefits of the Studio Environment

While the artists here primarily discuss the benefits of informal and social learning in the studio environment, they also recognize that these benefits are not experienced in a consistent way for all students. One artist describes "the downside" of the intensive studio environment in which he teaches, noting that the faculty teaching different sections are "trying to prepare students to give them the same kind of baseline skills, but it's pretty clear they don't all get that," so "whoever is in charge of teaching has a lot of responsibility," admitting that if a faculty member and student aren't a good match for each other, it affects the student's experience, and "I can see where some student complaints come from in that respect … it's so wonderful, though, in other ways if you can make it work."

Another artist mentions how first-generation college students can reportedly feel like second-class citizens at her current private college, some of whom arrive with less of an artistic skillset or experience base, so she and a colleague have designed intensive study-abroad trips for first-generation college students to give them an additional experience, noting proudly the success of a recent alum going to a prestigious graduate program.

One artist discusses how he sees more issues of privilege and bias with each year he teaches, pointing out how students arrive "all over the map" with regards to skills and experiences, and noting that the inconsistencies of the studio environment can be particularly problematic for students coming into the program with less preparation. In addition, when he began teaching at his current school, he was told to give more work than was physically possible to accomplish, contributing to a highly demanding and intense environment, and while that works for some students, he disagrees with that approach now, which resonates with Artist Two's comments about having a healthy studio environment rather than one that enforces bad habits through excessive pressure; here the suggestion is that this pressure could even become exclusive.

Gender certainly played a role in who benefitted from the community spaces of Artist Five's graduate program, and she recounts male faculty sleeping with female students, male faculty gathering to socialize, "always around drinking" and occasionally exchanging punches, and male faculty asserting that there were no good women artists. She offers these points saying, "it's so boring to talk about now," noting how much has changed in the tolerance for those sorts of behaviors. She also mentions that she had wonderful male peers in graduate school who didn't have to deal with the misogyny and sexism of the program the in same way as the women. Clearly, the program environment was not universally supportive, and Artist Five describes her peer group not as one type of social support for her learning, but as the majority of it.

Connecting Themes Across Disciplines

In-Group and Out-Group: Academic Difference

Looking across disciplines, throughout their education, some of these professors found themselves outside of the social center of the program, which was often related to the academic center. Earth Three recalls how he was not part of the "in-group" in his graduate program because his research wasn't closely related to that of others. Sometimes academic conversations among peers would happen during activities like basketball games, which didn't line up with his interests, but it didn't affect him much academically because the differences in research interests resulted in "a sort of casual exclusion." Interestingly, after graduate school he found himself in a research environment of professionals who had personal lives away from work. Because of that, work conversations happened at work, and the work environment was therefore "less social but more equitable."

Physics Seven is familiar with a few astronomy and physics departments and made the point that astronomy departments are often smaller and more narrowly focused than physics departments. He suggests that the astronomy departments tend to be more socially cohesive since their academic interests are closely related, so there is more likely to be the potential for an in-group and out-group. Conversely, there is less likely to be an out-group in physics because there is less likely to be an in-group formed by one main academic interest.

Academic differences can also become epistemological dividers between scholars. One scientist discusses how planetary science draws people from many different academic backgrounds, and "in terms of how it impacts our science …whether people are outwardly having the conversation or argument or not, I feel like we really frequently are arguing with each other about what it means to *know* something." She describes:

> … we sometimes end up with these culture clashes, where some folks [say], "Oh, you could never answer that question without having held those rocks in your hand," whereas other folks [say], "No, of course you can answer that question. If you derive this from the first principles of physics, you don't need to have ever seen the rocks" … and so I think that plays out socially … I think we do often have a lot of issues with… the *in* folks versus the *out* folks.

None of the artists interviewed here found themselves outside the social center of their academic studio environments for academic reasons, and in Artist Four's case it is clear that even strong disagreements about art did not interrupt the social cohesion and support of the studio. Perhaps this is due to the flexibility of art as a construct and the transparent way in which it is socially-constructed. Because there are many different views of knowing through art, artistic difference is to be expected, and this might invite more diverse perspectives to the studio. But it is worth recalling that one artist perceived one potential graduate program to be less socially supportive than another and chose not to attend, so there is also the possibility that the accounts here reflect the experiences of those who found the support to succeed professionally as faculty.

Gender and Isolation

Sometimes the “in folks” and “out folks” seem to be determined by factors other than academic perspectives, and gender was a factor across disciplines. Sexism in the studio environment made a significant impact on the experience of Artist Five, such that she relied more on a peer group than the faculty, placing her education even more in the informal spaces of the studio. Artist Four discussed more recent experiences, dressing extra-professionally to try to counter some of the pushback she felt she was receiving as a young woman faculty member, which may support conversations about how the additional labor of navigating bias affects the mentorship bandwidth of out-group faculty. Similarly, one scientist recounts being the only woman in her department and the relief she felt when another woman was hired, so there was someone else the students felt comfortable going to, describing the amount of tissues she would go through with crying students who felt she was the only one they could talk to. And another participant describes her present-day experiences, “There are still times where it upsets me and I feel like I don't belong,” saying there are still moments when she doesn’t want to adapt to be part of it. She continues:

> … I think I've also come to terms with … I'm here, and I've made it out the other end, and I'm going to make it better for others … some of it is confidence. I can do it, and I can do it my own way. And sometimes it stings, and it's a little bit

> hurtful, but I've learned to sort of brush aside comments or attitudes or moments where it's hard. That's sort of how I've tried to navigate the situation …

Women and men in the sciences discussed the experiences of women and the still male-dominant disciplines of field geology and physics. One geologist talks about how she sees the field as "awkward clubs," and in her own graduate program, the main club was a number of male peers. "In some ways I was not going to be part of that." She recalls how another female student would occasionally enroll in the same class but then drop it, later explaining that it was due to social discomfort. Another geologist describes feeling like she didn't have much of a community within her graduate program, even when on site in the lab, noting that her discipline was still dominated by men. "[F]or the first three years of graduate school … it was hard for me." She says of her graduate school program, "Academia is very clubby in general, and I felt like my experience was especially clubby in a lot of ways, and it continues to be … Field geology is one of these sub-disciplines that's very much an old boys club."

Meanwhile, one senior-career geologist has had mostly female PhD advisees for some years, noting that they might be even more passionate about fieldwork than his male students, and he recounts with excitement how successfully they are moving into the profession and changing the gender ratio, contributing to a turning point of gender equity in their sub-discipline, which resonates with a point made by a female geologist who notes that it is now getting better, as she mentions wonderful colleagues and mentors. The male geologist mentioned above attributes his success advising women in the field to honest conversations about any gender experience with which they are not comfortable. "I try to cultivate that degree of candor early on so that I really feel like if they had any impressions that were not favorable … that they would feel comfortable telling me about it, and I think I've been pretty successful in creating that degree of trust."

One male physicist acknowledges that he benefitted from a "level of unconscious comfort" with the work culture he experienced, describing a set of personality traits during his student years that lined up with a competitive, intense environment and mentioning how he now encourages certain students to go home because the cultural expectation is to work many long hours. Clearly a nearly nonstop presence in a competitive environment would privilege those who have the inclination and time to comfortably situate in that social environment. This physicist admits that most of the members of his undergraduate, graduate, and present research groups are white men. Although the group he runs now is slightly more diverse, women are few, and the group is "not doing better" than the typical "25 percent" women in physics.[1] He expresses disappointment that a successful female advisee recently left the group, preferring a different kind of mentorship style.

Another male physicist says, "There's a specific way we think about physics problems in the physical world that is acquired through experience." He continues that "in many departments there is a lot of learning that happens amongst the students just informally in social settings, and if somebody does not fit in socially, that can definitely have a detrimental impact," elaborating that when women are minoritized to the point of being a small fraction or single member of a research group, that can create exclusion that

[1] Porter and Ivie (2019) found this number to be 20 percent at the graduate level and 21 percent at the undergraduate level in 2017; astronomy was 33 and 40 percent, respectively. They note little movement in these numbers over the previous decade.

affects her experiential learning. In this way, being part of the social out-group may put more pressure on self-teaching or personal resourcefulness rather than informally letting the knowledge of the process, as one scientist put it, "filter in."

Discussion

The accounts of these artists and scientists start to tell the story of how their practices evolved in the social arenas of their disciplines and how they now oversee students in similar spaces. The studio, field, and lab were places of shared curiosity, building on intrinsic motivation with others and being around people who agree that their pursuits were worth their time. Peers entertained each others' whims and encouraged their wonder. It was their shared curiosity that brought them together in these spaces of practice, and here, they informally and experientially developed their processes.

The shared curiosity motivating the creation of new knowledge in these spaces varies in its focus and texture across disciplines, which affects the social environments. The scientists share a curiosity about something specific: the Earth, the stars, the ocean. The artists share a *type of questing* based on curiosity more generally. It is a way of observing, questioning, and exploring, and as Artist Two describes, a way of being in the world that is open and curious. As Borgdorff (2011) describes, the artistic process is a way of knowing, and as such, it can applied to anything, whereas the sciences have their ways of knowing, but the scientists here apply those ways of knowing to the specific content of their sub-discipline.

The studio environment, ideally, is supportive of individual artistic research addressing a great many ideas, and of openness and curiosity as a way of engaging generally, whereas the lab and field environments ideally support a focus on a more central set of concepts and methods. In the field, learning the way of seeing and reading the land requires students to, as one geologist says, “put in the time.” A beneficial field environment, then, is one that is inclusive and comfortable during an immersive experience as scientists and students rigorously address their curiosities about the Earth. The lab environments described here seem to place importance on inclusive cooperation and peer-learning that invites the contributions of lab members with diverse perspectives.

Relationships were key to learning in the social environments of the field, lab, and studio, as undergraduate and graduate students developed their processes through formal and informal mentorship and peer-learning. The more helpful accounts of mentorship were characterized by individual attention, responsiveness, and a recognition and appreciation of individual difference. For instance, Earth Seven’s advisor would deliberately encourage her to find her own way to approach a question. Similarly, Artist Two is careful not to influence students’ choices with his own judgments. Where peer-learning and middle-mentorship were primary ways of experiencing and developing the process, these important relationships were influenced by social comfort in the spaces, as One physicist discusses familial middle-mentorship in the lab and another physicist talks about how students share lunch, coffee, beer, and movies, while discussing physics. Most of the artists discuss the support they received from studio peers while working on independent projects in the studio, and Artist One discusses how just meeting peers with an interest in the same artistic form is a big part of the learning for his students.

As students, these faculty experienced a shared sense of curiosity and work ethic with others in their environment. The social resources there helped them to glean from their environments what they needed to succeed in their disciplines. They shared a love of exploring in the field, a glee about new data, an excitement about collaborations, a wonder at the sounds around them, and an appreciation for living in curiosity and awareness. In short, they shared an intrinsic motivation to be in the space, which created a sense of belonging. Time in the studio, lab, and field was essential to learning the process.

Where learning is informal and relational, social biases and privilege can play a significant role, seen here particularly with regards to gender. For instance, one artist was understandably extremely uncomfortable with the sexist environment of her graduate program, one scientist found alternative communities and collaborators to work with and learn from when gender was a barrier, and another scientist recounts getting more out of her field experiences with a female faculty member at the lead than when she was with competitive male peers. Notably, three women discussed drinking in relation to sexist environments, which questions the already-critiqued tradition of academic bonding over alcohol as an exclusive cultural practice. And these are the narratives are of women who stayed in the disciplines, naturally omitting the accounts of those who may have intentionally left due to bias or isolation, or who may not have accrued the social capital so helpful in navigating next steps.

Because the processes are imparted informally and socially in the studio, field, and lab, any bias or social factor that interrupts the comfort of students in the space then also interrupts their learning of the process– and the equity and inclusivity of these socially-constructed disciplines. Research shows that students from underrepresented backgrounds can feel excluded from these spaces of learning, practice, and important social connection (i.e., Núñez, Posselt, et al., 2019; Perez, 2016; Gonsalves, 2018). And notably, several of the sub-disciplines here have been slow to diversify in terms of gender and race, which lends additional significance to these few accounts.

Conclusion

In the studio, field, and lab, these faculty learned ways of knowing and refined aesthetic sensibilities. They entertained their whims and trusted their curiosities to create meaningful work. They taught themselves and each other, and they became authors in their disciplinary conversations, allowing their curiosities to guide their contributions. As higher education is changing, it is important to hold onto what is working well in these spaces– a sense of community around shared curiosity and relational learning from mentors, middle-mentors, and peers– while deliberately progressing towards greater inclusivity such that everyone is equally entitled to pursue their curiosities and co-author the conversations and spaces where those curiosities are explored.

The responsive mentorship in these disciplines imparts more than a standardized curriculum could contain, because it models dealing with unknowns and with cultivating and navigating social relationships. While biased behaviors certainly need to be addressed, greater inclusivity does not call for a standardization of the social spaces of the disciplines, where so much is learned informally, experientially–and enjoyably. Instead, these accounts suggest that a more reflective and transparently inclusive personalization of mentorship and peer resources could affect the same benefit to the "out-group" as to the "in-group," making these divisions unnecessary, such that the invitation to engage in the social environments of the discipline reflects a greater, shared curiosity.

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Discussion: Connecting Themes and Theories

Summary of Main Findings

This dissertation addressed the broad question of how the creative knowledge-generation process is imparted and developed in higher education, specifically in the curiosity-driven disciplines of pure mathematics, the physical sciences, and fine art. First, I compared the creative processes of pure mathematics and fine art, finding that the processes of these disciplines involve similar behaviors of inquiry, investigation, reflection, and revision; engage similar dispositions of openness and perseverance; rest similarly on disciplinary aesthetics; and respond to the social construction of their disciplines, asking then why such similar processes seen and taught differently. Then, I reported on a few main themes that emerged in interviews with mathematics faculty members, describing how question-finding in mathematics is viewed as a challenging part of the process and is learned and practiced socially, noting the role of mentorship, peers, the greater mathematical community, aesthetics, and social capital. Finally, I discussed the ways the creative knowledge-generation processes in fine art and the physical sciences are imparted in the social environments of the lab, field, and studio, finding disciplinary differences and a relationship between informal learning and the possibilities of in-groups and out-groups, particularly noting issues of gender. Throughout these three papers, this research highlights the interaction between the curiosity and wonder of the individual, the social construction of the disciplines, and the informal and social ways of learning to engage in creative knowledge generation. Additionally, affect, intrinsic motivation, and social and cultural capital played an important role in choosing and then engaging in the processes of these curiosity-driven disciplines.

This investigation is contextualized in a conversation about the value of curiosity-driven pursuits and the engagement in the creation of new knowledge as they relate to the aims of liberal education, particularly when intrinsically motivated by curiosity and wonder. Where “doing and undergoing” per Dewey (1934/1980) are key to the process and are guided by intrinsic curiosity, the dispositions of openness and perseverance are engaged, and the individual is transformed by attuned perception, awareness, and connection with the world, such that the aims of a liberal education are addressed. In this research, Dewey’s depiction of doing and undergoing fits well to the investigative processes of most of the interview participants, and to the artists and mathematicians cited in the first paper, though it is a stronger fit to the processes of those practitioners who engage directly in inquiry and find their own questions.

The Process is Imparted Informally

The faculty interviewed here told the stories of how they developed their own process and the ways they teach others. In most cases, there was no formula or curriculum; they described responsive mentorship, social learning, and self-teaching. Their relationships, and potentially their backgrounds, filtered the ways they absorbed or figured out the process. Early academic resources, artistic experiences, and educated parents featured in the earlier chapters of the stories of many, but not all, of the faculty, which may have provided exposure that could have attracted some of the faculty to their disciplines. Peer learning and middle-mentorship factored significantly in the ways these faculty members learned, offering them collaborators, support systems, and shared excitement for their wonder-driven investigations; as faculty members, many of them see to it that supportive peer environments are available to their students. In this way, the traditions of learning in the disciplines are selectively brought forward, and selectively changed.

Faculty advisors played a major role in the development of the processes for most of the mathematicians and scientists, with a few self-directed exceptions. Where they had good advisory experiences in graduate school, their faculty mentors are attributed with providing helpful big-picture advice, leading them to social connections, choosing good questions for them to address, or directly mentoring them as apprentices. In the few instances where they didn't have good experiences, or where they sought mentorship or role models elsewhere, their advisors are described as rarely present or nonresponsive, which may indicate the importance of personal fit between mentor and mentee such that communication styles merge well, or it may indicate the need for faculty to be proactive and flexible in communication style to serve diverse advisees.

These participants' experiences of advising and mentoring when they were students now influence the way they choose to mentor advisees of their own, either by repeating or changing what did or didn't work for them. When asked how they help students to develop their processes, they mentioned paying attention to what the students want to do and what they are drawn towards, and helping them to find their own way. Some of them extended this to fostering students' trust in their own ideas. Alternatively, faculty mentioned onboarding students to the research process via provided questions or small projects that were parts of a larger one, such that the students could gradually develop autonomy as practitioners themselves.

In addition, autodidaction played no small part for the interviewees. They taught themselves art histories, mathematical theorems, natural history, scientific methods, art techniques, coding, even mathematical proving–alone. Successful creative authorship in these fields requires the ability to be self-directed in finding and exploring open-ended questions or complex ideas; they also exercised the socio-emotional dispositions required to do so. Whether their success in their fields and their eagerness to self-teach are both driven by curiosity, or their self-teaching was necessary for the pursuit of their curiosity, they were highly motivated to learn.

The Process is Practiced and Experienced Differently Across Disciplines

The creative knowledge-generation process was practiced and experienced quite differently across the disciplines. Some practitioners engaged directly in very immediate iterations of inquiry, investigation, reflection, revision, while others were directly engaged in only a subset of these behaviors, either because the work extended across a long timeline, or because it was distributed among team members. Although the process of knowledge-creation in each discipline seemed similar, the role and experience of the individual practitioners varied, and as such, the different parts of the process were observable and imparted to students.

Learning Inquiry

Inquiry is a foundational part of creative knowledge generation, but early engagement in self-guided inquiry was more consistently present in the experiences of the artists than in the experiences of the mathematicians or scientists. Inquiry behaviors were supported socially in the art studios by "entertaining whims," giving feedback, exchanging ideas, and just being there, together, as each person authored their own questions in their process. The studio itself is described in the first paper as a place of inquiry (i.e., Gude, 2004, 2007; Shields, 2018; Sjöholm, 2013; Sullivan, 2010). The roots of this artistic inquiry were often planted early, by independent artistic pursuits, such that the artists generally discuss arriving at independence with regards to idea generation by the middle of their undergraduate experiences. With their own students, they often supported inquiry behaviors by *not* giving pointed feedback, as with Artist One, Artist Three, and Artist Four, who instead prioritized individualized encouragement and promoted self-trust, which resonates with Shields' and Gude's description of the art studio as a place for not-knowing.

Idea generation or question-finding happened much later for mathematicians, and this skill was gradually imparted socially, via mentorship and modeling. Although the accounts of mathematicians in the first paper assert the importance of mathematical inquiry across the curriculum and assert that mathematical creativity lies in the interplay between problem-posing and problem-solving (i.e., Silver, 1997), many of the mathematicians interviewed discussed question-finding (or problem-posing) as the hardest part of the work of a mathematician, and they were often not prepared to independently take on this task until after receiving their PhD. In most cases, their graduate advisors provided research questions to them and mentored them to gain entry to the social field of mathematics, where other questions often arose, so they could avail themselves of that resource after graduate school. Now, as faculty, they endeavor to do the same with their advisees.

The scientists learned by doing, in some cases "being thrown into the deep end," often by joining a project in process, thereby learning inquiry experientially and through modeling. Most of the scientists mentioned that they now provide the question or project to their current students and model the process of investigating it for them, either by working together or in conversations and group meetings, which is similar to how they learned. Because the processes across the sciences vary with regards to the length of a given investigation, logistical preparation, interactions of collaborators, and connections to a greater disciplinary initiative, the work of question formation had often been completed by another scientist, perhaps at a different institution or at a time before a student's (or a faculty member's) arrival. Some of the lab scientists in particular had developed processes that seemed to engage more problem-solving than problem-finding, but other scientists hiked out to the wilderness or looked to the sky, "fishing" for a landform or phenomenon to investigate and building a research agenda through their own interests.

Because of the very different ways that idea generation and question-finding played out in the disciplines themselves, at least in higher education settings, they were presented differently in the training of a student and were experienced at different moments in their education. It would be fair to say that some of the interviewed faculty members had been exposed to a scientific process but not the "full" creative process during their education, at least not at the level of the individual, while the mathematicians eventually were, and the artists had already experienced the full creative process, inclusive of authorship of the question, during their undergraduate studies.

Investigation and Exploration

Investigative behaviors were also imparted differently across the disciplines. In collaborative scientific processes or on science teams, investigation, reflection, and revision were conducted by multiple different "actors" and at different times, resulting in variation in the informal ways the processes were imparted and the timing of when each behavior was practiced. Meanwhile, the investigative processes described in the interviews with mathematicians and artists seemed notably similar, which was unsurprising given the comparisons mathematicians have long made between themselves and artists, each sometimes involving an intellectual or literal meandering, independently and iteratively trying different approaches, attentively working to realize or express abstract ideas, or finding someone with whom to share or exchange ideas. As the mathematicians worked things out in drawings, in code, on a board, or on paper, the artists similarly emphasized figuring out an idea via making.

Wonder

The practitioner accounts describe a durational and challenging process guided by intrinsic motivation, curiosity, and wonder. Their pursuits involve some version of stick-with-it-ness, whether perseverance or patience, in addition to openness or the humility to change a question or approach when necessary. All of the interviewees, unsurprisingly given the accomplishments that led them to tenure-track positions, either discussed or depicted an intense work ethic, usually paired with a great appreciation for the process. Math One states multiple times that, "The important thing is you have to enjoy the process… That's the bottom line," and Earth Two describes his path as "go in the direction of fun." One planetary scientist mentions how members of her community love when they are wrong, detailing the joy of her own group and her colleagues in finding out that a planet was "even weirder than they thought." Most of the artists talk about how much they enjoy the social camaraderie of the studio environment and the ongoing motivation to explore their ideas in the studio, and Artist Five says of pursuing a new idea, “That’s the excitement, right? The discovery…” Math Four tells how, “I found something that I love,” as she describes working long hours to find patterns. Physics Four says, “Once I realized how big the universe was … I couldn’t imagine doing anything else,” and Earth Five says of rigorous fieldwork experiences, “I love this. I wouldn’t do anything else.”

They love the search, the discovery, the striving, the quest. And as authors within their disciplinary communities, they forge the directions of the discipline’s conversations through the questions they choose to ask, methodologies they use (and invent), colleagues they cite, and collaborators they seek. The evaluative criteria of these choices include a resonance with the discipline’s aesthetic values and the questions it deems interesting. These disciplines are then built on the cumulative curiosities of those practitioners in positions of authorship. And collectively, the accounts here resound with a texture of inspiration and awe. These practitioners are inspired and motivated by wonder at the phenomena they investigate, and the disciplines evolve to reflect the questions arising from this affective state: They are *wonder-constructed* disciplines.

Capital

Throughout their education, students accrue cultural capital in the form of knowledge and values, and social capital in the form of connections, which together will help them to move forward in the discipline. The social capital and cultural capital imparted in training builds, first of all, on the cultural capital necessary to join: an awareness of and appreciation for the aesthetics of the discipline that lines up with a student’s intrinsic motivation. Then this shared aesthetic sense imparts more social capital in the form of contacts with similar curiosities, and more exposure through which to gain cultural capital in the form of refined disciplinary knowledge and aesthetic sensibilities, which could be a deeper understanding of a the criteria for a good question or what is cool and beautiful.

The disciplines are often created, therefore, by those who fit well to the academic and social center of their graduate school environments where social capital gets built, which is also based on an alignment of shared aesthetic values, a form of cultural capital, and a common enthusiasm for their disciplines. They find camaraderie by sharing intrinsic motivation, wonder, and curiosity. It is therefore important that a students' affect and aesthetic sensibilities place them in a discipline such that their intrinsic motivators line up with the motivators of the people who are already there, the accomplished contributors and authors of the contemporary conversation, who are also the curators of the discipline. Background cultural capital may play a role here, such that a student has exposure and access to a discipline that aligns with their sensibilities– and that they are entitled to pursue it. But in some ways, sheer enthusiasm may act as "starter" capital.

Theoretical Summary

The way students learn to engage in the creative knowledge-generation process can be seen as the gaining of social resources and the building and refinement of knowledge and aesthetic values through practice and observation. The shared enthusiasm and aesthetic sensibilities students bring to their informal learning environments create the connections they need to participate in disciplinary conversations, and as they refine their knowledge and build relationships, they are invited to engage and become entitled to contribute to the disciplinary conversation that resonates with their curiosities and sensibilities. The way the process is imparted, then, can also be seen as the way an invitation is extended to engage in this process, and through this lens, the accounts in this research describe how that invitation is extended and pursued. Typically through mentorship and practice, students are afforded opportunities to develop skills, knowledge, and connections such that they can eventually independently engage in the creative knowledge-generation process of their discipline and author new knowledge based on their curiosities.

The creative knowledge-generation process that most of the interviewees discuss, and that mathematicians and artists describe in the reviewed literature, is an ongoing, iterative process by which knowledge or awareness is generated, and while it kicks out a product, a proof, a paper, an artwork, or a composition periodically, that does not necessarily indicate the immediate goal of the process, because it is a continual experience of exploration, which resonates with Dewey's interpretation of doing and undergoing. Although some of the practitioners do seek to eventually achieve a particular task, this more exploratory texture of the process is particularly true of the artists' accounts, and some mathematicians and scientists similarly discussed the trust they have in their processes to eventually lead them to a result, such that they typically can focus on inquiry and exploration.

Placing Dewey's description of the iterative and ongoing creative encounter in conversation with Bourdieu's (1986) notion of interrelated social and cultural capital situates Dewey's doing and undergoing relationally and provides a way to regard how social and cultural factors facilitate engagement in creative knowledge-generation processes. By extending Bourdieu's conceptualization of social and cultural capital from general terms to discipline-specific forms, we can describe some of the experiences shared through this research. Disciplinary social capital can be seen as social connections and support within the discipline that facilitate engagement, and disciplinary cultural capital, which takes time and experience to build, can be seen as sharing in the aesthetic values, epistemology, and the canon of the discipline. In Bourdieu's model, the accrual of one of these types of capital helps the other accrue, and it seems from the interviews that disciplinary canonical knowledge, aesthetic sensibilities, and social connections similarly lend to the accumulation of each other.

The resulting heuristic model contextualizes Dewey's description of the transformative, creative encounter with Bourdieu's notion of social and cultural capital (see Figure 1). On one side, the discipline is situated in the broader culture. Disciplinary cultural capital is imparted through experience, with exposure to mathematical proofs of elegance, to exhibitions and lectures, to the choices and processes of mentors. The knowledge and values that comprise this capital elaborate upon intrinsic motivators that, it seems, attract practitioners in these disciplines to their pursuits to begin with: the aesthetic sensibilities and shared curiosity of the discipline. On the other side, we see the social field of the discipline, which is situated in the broader social arena, so one's social capital affects interactions in the social field of the discipline and may affect "social fit." Social and cultural capital spiral up where one helps to accrue the other, and the compounding nature of social and cultural capital then frames the disciplinary social field and disciplinary canon as arenas into which entry is imparted via shared affect, curiosity, intrinsic motivation, and aesthetic sensibilities. In this way, the heuristic model relates intrinsic motivators to the social and disciplinary contexts of the process, and it clarifies how they afford entry to the discipline and the invitation to engage in it.

Considerations Moving Forward

This research draws a connection between the aims of liberal education and engagement in the creative knowledge-generation process, particularly when it includes question-formation. It is therefore helpful to consider where and how students engage in the full creative process, inclusive of moments of inquiry and idea generation, investigation, reflection, revision, and iterations that may alter the question or the method of investigation, because it is in this full process that students experience the *doing and undergoing*, the flexible thinking, shifting perspectives, sensitive perception, and complex ideas that link the creative knowledge-generation process to liberal education.

The faculty learned their processes informally, relationally, and often in the social spaces of the disciplines– and it was because of the informal nature of the learning that they were exposed to inquiry behaviors, ways of knowing, and aesthetic values. It is the personalized nature of the training that worked well. That being said, it is clearly important to endeavor to make this informal learning more equitable and inclusive. Furthermore, given the relationship between the pursuit of creative knowledge-generation in these curiosity-driven disciplines and the aims of a liberal education, an invitation to engage in the environments where the process unfolds is also an invitation to engage in the liberating intellectual behaviors of attuned perception, free thought, inquiry, and authorship.

This raises additional question about the way these curiosity-driven disciplines are imparted and developed over time, and whose curiosities they pursue. At the core is the purpose of higher education. Where this purpose is related to liberating intellectual and personal practices of inquiry and idea generation, durational investigation, and, as Artist Four explained, a trust in the value of one's own ideas, then pure mathematics, exploratory field sciences, curiosity-driven physics, and the fine arts are among the academic paths to these goals.

Accounting for Myself in the Data

It is important to mention my own subjectivities and my arts background influenced this research. I brought a particular understanding of artistic processes and contemporary art literature to these studies, which provided a helpful lens through which to comprehend the social construction and iterative nature of the processes across these disciplines. In the comparison of artistic and mathematical processes, the artistic process was my starting point from which the comparison then unfolded, eventually creating a rich enough picture of the mathematical creative process to use as a lens to look back at the artistic process. The participants were aware that I have an arts background based on a brief mention in my introductory email, and at times I sensed that our conversations may have been affected by this. The mathematicians and scientists may have discussed their processes differently with me than they would have with an interviewer from their own disciplines, while conversations with artists started at a different level, where they knew I knew where they were coming from, and I had to explicitly ask at times for basic information, explaining that we both, as artists, knew what we were talking about, but that it was helpful for me to check my assumption and to have an idea described in the data.

I am certain that the data I received about gender was influenced by my own. There were moments when I shared in the feelings of the women I interviewed, and the conversations had a texture that indicated a common understanding. I suspect that the conversations about gender with men were affected, too, though I feel that the degree to which this is true varies with regards to each participant's own comfort and experience discussing gender; most of the conversations reflected a good deal of previous thought and attention to these issues.

Finally, as this research is exploratory and interdisciplinary, there were many directions each conversation, literature search, or model could follow. I listened to the voices in the literature and the perspectives of interview participants, which at times expressed that our best work arises when we follow our curiosities and trust that they will lead us to something meaningful. For this reason, the paths through the conversations, the data, and the literatures are unique, particularly because my own affinities for curiosity-driven disciplines and my interests in the aims of liberal education and issues of gender highlighted certain moments in each conversation and each reading, and following those moments like an artist in the studio, this research reflects my own sense of curiosity.

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Figure 1.

Doing and undergoing contextualized by social and cultural capital.